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Comparative Analysis of Classical and Intelligent Control Techniques for the Inverted Pendulum System: Design, Simulation, and Performance Evaluation

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***Abstract*—** The inverted pendulum, a classic problem in control theory, is widely used for analyzing nonlinear systems and potential energy applications. This study investigates various controller techniques classical and intelligent applied to the inverted pendulum system, providing a comprehensive review of their design, performance, and limitations through simulations and graphical analysis. Key control strategies, including PID, Linear Quadratic Regulator (LQR), Neural Networks (NN), and Fuzzy Logic Control (FLC), are implemented and evaluated for system stability, response time, and robustness under structural uncertainties and external disturbances.Simulation results reveal that classical controllers, particularly the PID and LQR, demonstrate superior performance in terms of settling time, overshoot, and steady-state error under linearized system assumptions. However, these conventional techniques struggle to handle nonlinearities and uncertainties effectively. To address these limitations, robust controllers such as the LQR are employed, minimizing actuator effort and optimizing cost functions. In contrast, intelligent controllers, including NN and FLC, exhibit adaptive capabilities, are enabling them to handle complex, nonlinear dynamics with greater accuracy and reliability. NN and FLC outperform classical controllers in terms of faster response times, improved stability, and reduced computational cost. This study underscores the advantages of NN and FLC in achieving enhanced system performance while maintaining stability and trajectory tracking for both cart position and pendulum angle. MATLAB/Simulink simulations validate the efficacy of each control strategy and highlight the potential of intelligent control techniques in addressing challenges associated with nonlinear systems and robotics technology. The findings provide critical insights into the design and application of control strategies for inverted pendulum systems, with implications for advancing feedback control in robotic systems.

***Keywords*—** Inverted Pendulum, Control Techniques, Neural Networks (NN), Fuzzy Logic Control (FLC), MATLAB/Simulink Simulations

I. INTRODUCTION

The inverted pendulum (IP) is a classical control problem, consisting of a pendulum attached to a cart. In its default state, without input, the pendulum stabilizes in a downward position [1]. The system is inherently unstable due to the centre of mass being above the pivot. Maintaining balance requires continuous control, as the system is highly sensitive to disturbances [2]. Model uncertainties and transmission errors contribute to the system's instability, making it a challenging yet valuable test case for control techniques [3]. The inverted pendulum system is highly unstable, with a Single Input and Multiple Output (SIMO) configuration, where input is applied via force or torque, and outputs include cart position and pendulum angle [4]. A nonlinear mathematical model is derived using Newton's laws to balance forces in the x and y directions, followed by linearization for analysis under multiple input parameters [5]. Both linear and nonlinear control techniques are applied to evaluate the system's behaviour and response.

The inverted pendulum system is characterized by two equilibrium states: stable, with the pendulum downward, and unstable, with the pendulum swinging upward [6]. Due to its nonlinear dynamics, small disturbances cause instability, necessitating control to maintain balance [7]. The system operates as a SIMO configuration, with cart position and pendulum angle as outputs and applied force as the input [8]. The inverted pendulum (IP) is a highly nonlinear and inherently unstable mechanical system with limited degrees of freedom [9]. Its complexity makes it a standard benchmark for designing, testing, and comparing both conventional and modern control systems [10]. Due to its instability, the inverted pendulum remains a challenging problem in control engineering research.

Intelligent systems, including robotics, mechatronics, and control systems, have gained prominence in technical applications, driving research into intelligent algorithms [11]. Key intelligent control methods, such as genetic algorithms, fuzzy logic, neural networks, and reinforcement learning, are utilized for controlling dynamic systems [12]. Neural networks and fuzzy logic are particularly favored in real-time control applications due to their adaptability and learning capabilities [13]. Fuzzy logic effectively translates human intuition into numerical representations, though optimizing fuzzy rules for systems can be challenging. Genetic algorithms provide powerful off-line optimization but are unsuitable for real-time control due to lengthy iterative processes [14]. Combining intelligent tools, such as neuro-fuzzy controllers, integrates neural networks with fuzzy logic, enhancing system performance and adaptability [15].

The inverted pendulum is a fundamental study subject for dynamic model analysis and control strategy development due to its highly nonlinear and inherently unstable nature [16]. The primary challenge lies in maintaining vertical stability while ensuring robust and stable closed-loop performance with simplified control algorithms [17]. Its relevance spans various fields, including robotics, power systems, and industrial processes, exemplifying complex defiance control problems [18].

The system consists of a cart at zero position with a downward-facing pendulum, where an applied force induces oscillatory motion in both the cart and pendulum due to Newton's laws of motion [19]. The force applied to the cart causes displacement, while the pendulum resists and adjusts its angle, leading to to-and-fro motion [19]. Horizontal torque applied to the cart generates sufficient force to stabilize both translational and rotational movements, ensuring smooth system operation [20]. The Inverted Pendulum system, where the pendulum is stabilized at its unstable upward position, presents a complex control challenge [21]. It is crucial in applications requiring high precision, such as rocket missile projection control and self-balancing technologies like Segways and robots [22]. These systems demand advanced control techniques to achieve accurate balance, surpassing human and machine capabilities in dynamic environments. The Inverted Pendulum is a nonlinear, underactuated control system where classical controllers like PD and PID struggle with disturbances and uncertainties [23]. Fuzzy Logic Control (FLC) emerges as a robust alternative, combining the strengths of classical algorithms with heuristic expertise to handle nonlinear systems lacking precise mathematical models [24]. By utilizing error and error rate as inputs, FLC demonstrates superior adaptability and performance in complex control scenarios.

The PID controller, known for its simplicity and efficiency, remains the most widely used control technique in industrial systems due to its adaptability and ease of implementation, managing over 90% of low-level control tasks [25]. However, the Inverted Pendulum (IP) system, characterized by instability, higher-order dynamics, and nonlinear uncertainties, serves as a benchmark for testing advanced control strategies [26]. Its applications span robotics, space vehicles, guided missile systems, and surveillance technologies like Segways, showcasing its importance in both experimental and practical domains [27]. Optimal control of dynamical systems, both linear and nonlinear, has advanced through intelligent computational techniques like artificial neural networks (ANN), fuzzy logic (FL), and evolutionary algorithms (e.g., GA, PSO) [28]. These methods provide innovative solutions for challenging control problems, including the stabilization of complex systems like the inverted pendulum and quadrotors. Control strategies such as Lyapunov-based schemes, adaptive back-stepping, and optimal methods like LQG further demonstrate the applicability of advanced approaches in nonlinear system control and trajectory tracking [29].

While advanced control techniques like fuzzy logic, neural networks, and adaptive strategies offer intelligent automation, their complexity and computational demands often limit practical implementation. This study employs simple yet effective LQR and PID control methods to stabilize the inverted pendulum-cart system, analyzing its performance under disturbance and non-disturbance conditions. Our results demonstrate that these straightforward control approaches ensure efficiency, reliability, and ease of application for dynamic system control.

II. MATERIAL AND METHODS

In this study we employ a range of methods to analyze the control of inverted pendulum systems, focusing on both classical and intelligent strategies. Classical controllers, including Proportional-Integral-Derivative (PID) and Linear Quadratic Regulator (LQR), are implemented and evaluated for stability, settling time, overshoot, and steady-state error under linearized assumptions. Intelligent control techniques, such as Neural Networks (NN) and Fuzzy Logic Control (FLC), are utilized for their adaptive capabilities to handle nonlinear dynamics and uncertainties. MATLAB/Simulink is used for designing, simulating, and validating the control strategies, with graphical analysis comparing their performance. Robustness is tested under structural uncertainties and external disturbances, while LQR minimizes actuator effort through cost function optimization. NN and FLC demonstrate superior response times, stability, and reduced computational cost compared to classical controllers. Simulation results highlight the advantages of intelligent controllers in improving system performance. The study also provides insights into trajectory tracking for cart position and pendulum angle. These findings underscore the potential of intelligent control techniques for addressing nonlinear system challenges, particularly in robotics and automation.

A. *Mathmatical Model*

The mathematical modeling of the Inverted Pendulum (IP) system is derived using the Lagrangian or Newtonian approach [1]. The system consists of a cart moving freely along a frictionless rail, with a pendulum of specified length attached at the cart's center, rotating about the pivot point along the vertical axis. The dynamic equations governing the uncertain IP system are formulated based on these physical assumptions and system geometry [30].

$$\begin{bmatrix} x1 \\ x2 \end{bmatrix} = \begin{bmatrix} x2 \\ f(x1, x2) \end{bmatrix} + \Delta A \begin{bmatrix} x1 \\ x2 \end{bmatrix} + \begin{bmatrix} 0 \\ g1(x1) \end{bmatrix} \mu \quad (1)$$

$$f(x1,x2) = \left[gsin(x1) - \frac{mplx2\sin(x1)\cos(x1}{a}\right]/b, \quad (2)$$

$$g1(x1) = \cos(x1)\,/ab \quad (3)$$

$$a = mp + mc, b = \frac{4l}{3} - \frac{mplcos2(x1)}{a} \quad (4)$$

The inverted pendulum system parameters include pendulum angle X1, angular velocity X2, control input µ (torque in Newtons), cart mass, pendulum mass (in kg), pendulum length l, (in meters), and gravitational acceleration g=9.8m/s2. Structural uncertainty (ΔA) accounts for parametric variations such as friction, temperature, and force, which are modeled dynamically due to measurement imprecision and non-uniform influences. These uncertainties necessitate robust control strategies for system stability.

B. Implemented Control Technique

1) Linear Qudratic Regular Technique

The Linear Quadratic Regulator (LQR) is a classical control algorithm that provides optimal infinite gains by minimizing a defined cost function. The cost function, represented as a parabolic equation, ensures desired system characteristics by balancing state and control efforts. LQR is implemented to achieve system stability and optimal performance efficiently.

$$J = \int_0^{\infty}[x^T(t)Q(t)x(t) + U^T(t)R(t)U(t)]dt \quad (5)$$

Here, x(t) represents the different states where, U(t) represents the input function applied to the system Inverted Pendulum system in terms of force or torque. Q(t) and R(t) represent the weight matrix.

$$K= R^{-1}B^TP \quad (6)$$

$$K = lqr(A,B,Q,R) \quad (7)$$

Here, in above equation P represents stable poles for the system and gains of the system are calculated by The Riccati equation given in equation.

$$ATP + PA - PBR{-}1BTR + Q = 0 \quad (8)$$

The State Space (SS) representation of the linearized model is given as in equation

$$\dot{x} = Ax + Bu \quad (9)$$

By assigning the parameter values which is calculated from above state space model, and we get the following results as shown below.

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 53.6927 & 0 & 0 & 0.4454 \\ 0 & 0 & 0 & 1 \\ -8.0539 & 0 & 0 & -0.1481 \end{bmatrix} \quad (11)$$

$$B = \begin{bmatrix} 0 \\ -4.4543 \\ 0 \\ 1.4812 \end{bmatrix} \quad (12)$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \quad (13)$$

The LQR algorithm computes the optimal gain matrix K, by adjusting system poles within the stable region of the S-plane, ensuring system stability. Using the state-space representation, it linearly models the relationship between state variables and inputs to predict future states. Stability is verified via MATLAB by analyzing the eigenvalues of the A-matrix to confirm pole placement in the left half of the S-plane.

C. MATLAB Task

MATLAB is a high-level programming environment widely used for modeling, simulation, and analysis of dynamic systems [31]. It provides robust tools for implementing control algorithms, performing stability analysis, and visualizing system behaviour [32]. In this study, MATLAB is utilized for designing controllers, analyzing state-space models, and validating system stability through simulations.

$$\gg Poles = eig(A) \quad (14)$$

Poles = 0 7.2946-7.3614-0.0813 (15)

The positive pole 7.2946 is in the right plane which shows that the system is unstable position of the system in open loop response.

The control-ability of the system is calculated through the rank of system. By using MATLAB, we have find out the rank as given below.

$$\gg RankCM = rank(ctrb(A,B)) \quad (16)$$

>>Rank CM =4 (17)

The rank of the system is 4 which denote all its four states mean this system is controllable.

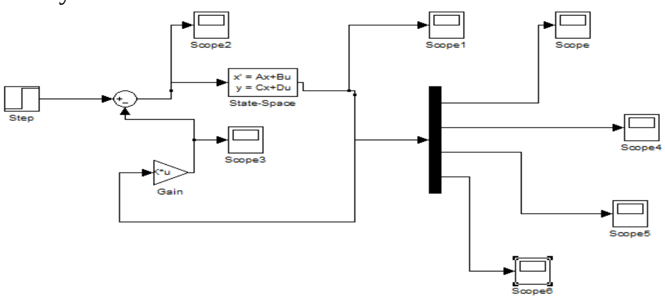

Figure 1Shows simulink model for the Linear Quadratic Regulator (LQR)

Controller, designed to stabilize the inverted pendulum system. The model integrates system dynamics and control parameters to optimize performance.

D. PID Controler

The PID controller, widely used in industrial and academic control systems, is effective for both linear and nonlinear systems due to its simplicity and ease of implementation. It operates using three control gains: Proportional (Kp), Integral (Ki), and Derivative (Kd), to optimize system performance.

$$PID = Kp + \frac{Ki}{s} + Kds \quad (18)$$

Using the calculated gains, a hit-and-trial method was employed to stabilize the cart and pendulum angle, minimizing steady-state error, rise time, settling time, and overshoot. The proportional gain (Kp) reduced oscillations, integral gain (Ki) eliminated steady-state error, and derivative gain (Kd) enhanced system stability and response speed.

E. Optimal ControUsing LRQ

Optimal control involves designing control strategies that optimize system performance based on a defined criterion, such as minimizing a cost function or maximizing performance [33]. It ensures the system adheres to physical constraints while achieving desired objectives, such as target tracking or trajectory following. The linear quadratic regulator (LQR) is a robust and efficient optimal control technique that minimizes a performance index by considering both control inputs and system states [34]. LQR provides a reliable solution for stabilizing dynamical systems under specified constraints.

The linear state-space equation is found as x 0 = [0, 0, 0, 0]T after linearizing nonlinear system equations about the upright (unstable) equilibrium position.

$$Ax+Bu\dot{x}=Ax+Bu \tag{19}$$

$$x=[\theta,\dot{\theta},x,\dot{x}]Tx=[\theta,\dot{\theta},x,\dot{x}]^T \tag{20}$$

Where
The state feedback control $u = -Kx$ leads to

$$\dot{x}= (A-BK)x\dot{x}=(A-BK)x \tag{21}$$

where K is derived from minimization of the cost function:

$$J=\int(xTQx+uTRu)dtJ=\int(xTQx+uTRu)dt \tag{22}$$

Where Q and R are positive semi-definite and positive definite symmetric constant matrices, respectively:
The LQR gain vector K is given by

$$K=R-1BTPK=R-1BTP \tag{23}$$

where P is a positive definite symmetric constant matrix obtained from the solution of matrix algebraic Riccati equation (ARE)

$$ATP+PA-PBR-1BTP+Q=0. \tag{24}$$

The instantaneous states of the nonlinear inverted pendulum system, including pendulum angle, angular velocity, cart position and velocity, are measured and fed into the LQR for optimal control. A linear state-space model is used to design the LQR, which is combined with the PID controller for enhanced control performance. The LQR control input is applied negatively to the PID control input for optimal control action. The PID and PID+LQR controllers are tuned using a trial-and-error process to achieve the desired system responses.

F. Fuzzy Logic Contrler

The Fuzzy Logic Controller (FLC) is a widely used control approach for complex systems in scientific and industrial applications due to its ability to handle non-linearity, uncertainties, and disturbances without requiring a complete mathematical model [35]. It is implemented in the inverted pendulum system for its simplicity and efficiency compared to conventional control techniques. FLC effectively manages systems with poorly defined dynamics or those lacking analytical representation, making it an ideal choice for challenging nonlinear systems. Unlike other nonlinear control methods that depend on predictable system behavior and well-defined parameters, FLC provides robust control for systems with uncertainties. It has proven to be more effective than traditional control algorithms in delivering precise results under complex and ill-defined conditions [36]. By addressing system complexities, FLC ensures stable and accurate control, making it a reliable option for handling dynamic and unpredictable behaviours.

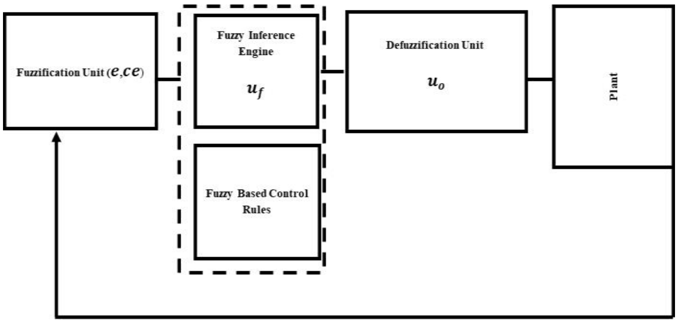

Figure 2 The fuzzy Logic Controller (FLC)

Its implementation for systems with complex, ill-defined, or analytically challenging models, providing more precise results compared to conventional control algorithms.

Where, e Is the Error, ce is Change of error, u_s is Output and u_o is Controller output. Main operation for FLC is given as.

1) Fuzzification Interface unit Fuzzy based control rules, Inference engine unit
2) Defuzzification interface unit.

Table 1Parameters for the simulated inverted pendulum

Symbol	Parameters	Weight
M	Cart mass	0.445kg
M	Pendulum mass	0.21kg
L	Center of mass and pendulum distance	0.305m
I	Moment of inertia	0.006kg*m^2

Table .1 present the system that is taken certain assumptions for this problem. Many parameters used for this problem for simulation purpose are used in Table .1.

III. RESULTS

The simulation and analysis of the PID controller, ANFIS, and Fuzzy Logic Controller, comparing their performance under identical conditions, including membership functions, rules, and scaling factors. The controllers were tested on three tasks: applying a step input to the cart's position transfer function and a discrete impulse to the pendulum's angle transfer function. Results highlight the performance differences across controllers using various tuning techniques.

A. PID Controler

The PID controller is designed to control the cart's position with a step input time of 2 seconds and a final set point value of 1 (Figure 1A). The controller compares the current cart position to the set point and generates an output to adjust the cart's movement toward the set point. This output is applied every 2 seconds (Figure 1B) until the set point is achieved. Proper tuning of the controller is essential, involving adjustments to the proportional, integral, and derivative gains to optimize performance. Once tuned, the controller continuously measures the position error and adjusts the output accordingly. The system ensures precise control, moving the cart to the setpoint within the specified time frame (Figure 1).

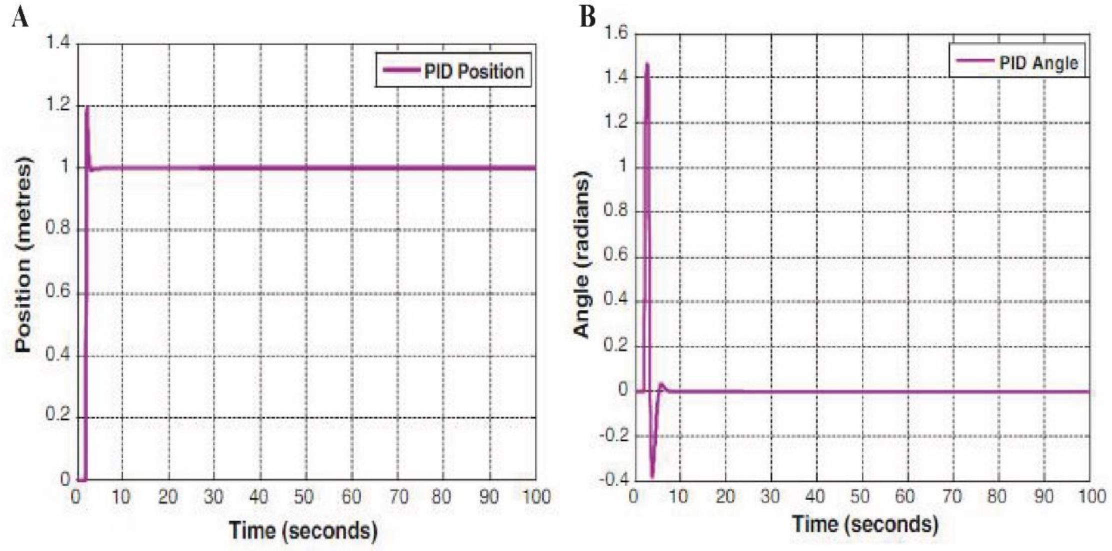

Figure 3 PID controller design for cart position tracking. (A) The controller responds to a step input with a setpoint value of 1, adjusting the cart's position every 2 seconds. (B) The controller design ensures accurate tracking and timely movement toward the desired setpoint, with the step input time set to 2 seconds and a final setpoint value of 1.

When the discrete impulse input delay is 2 and the final set point value is 0, the PID Controller responds to the angle of the pendulum.

B. Fuzzy Logic Cntroler

The Fuzzy Logic controller's response to the cart's position, with input step time set to 1 and final set point value set to 1 (Figure 2A). The Fuzzy Logic controller's response for the pendulum's angle with a discrete impulse input delay is 2, and the final set point value is 0. The Fuzzy Logic controller's response to the cart's position, with an input step time set to 1 and a final set point value set to 1, is shown in (Figure 2). Figure 2A displays the cart's position at the start of the simulation while Figure 2B shows the cart's position after the Fuzzy Logic controller has been implemented. It can be seen that the cart's position is shifted towards the target value of 1 over the course of the simulation, indicating that the Fuzzy Logic controller is able to effectively direct the cart towards the desired set point. This demonstrates the robustness of the Fuzzy Logic controller in responding to changing conditions with a set input step time and set point value.

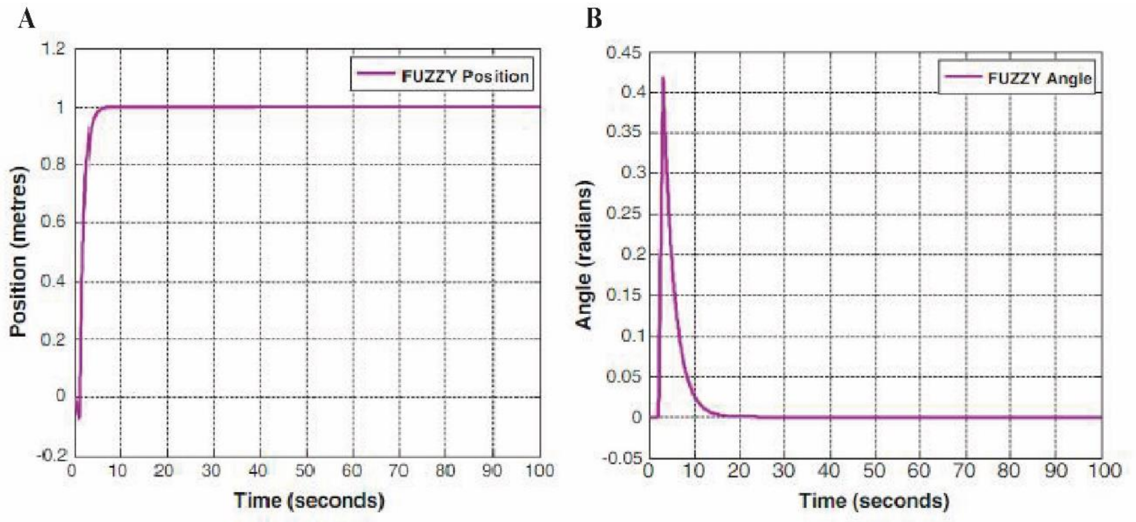

Figure 4 Fuzzy Logic Controller responses for cart position and pendulum angle. (A) Step response for cart position with a set point of 1 and (B) step response for pendulum angle with a set point of 0, both with an impulse input delay of 2.

C. ANFIS Controler

The response of the ANFIS controller for cart position tracking, with an input step time of 1 and a final setpoint value of 1, is illustrated in (Figures 3A and 3B). The controller effectively directs the cart to the desired setpoint, demonstrating precise and stable performance. Additionally, the ANFIS controller's response to the pendulum angle, with a delayed impulse input of 2 and a final setpoint value of 0, highlights its capability to handle dynamic inputs and maintain stability, achieving a response time of 2 seconds.

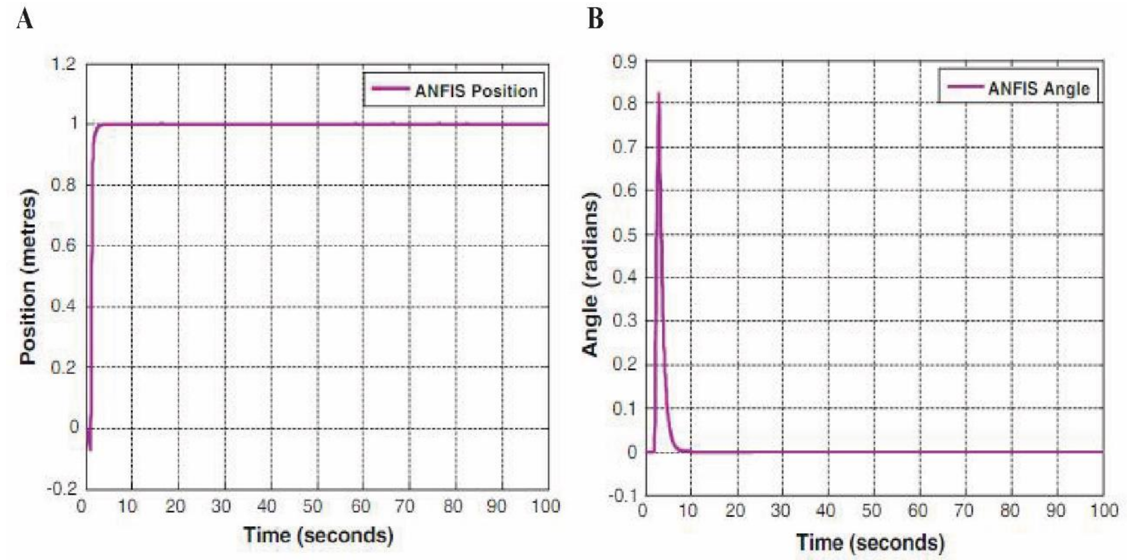

Figure 5 Responses of the ANFIS controller. (A) Step response for cart position with an input step time of 1 and a final set point value of 1. (B) Discrete impulse response for pendulum angle with a delayed impulse input of 2 and a final set point value of 0.

Table 2 presents a comparative analysis of performance metrics for various approaches, including ANFIS and conventional control methods. Key performance indicators such as settling time, overshoot, steady-state error, and adaptability are evaluated. The results indicate the superior efficiency, precision, and robustness of the ANFIS controller over conventional methods, emphasizing its suitability for nonlinear systems and complex dynamic scenarios.

Table 2Comparison among two methods for tuning a PID controller

Controllers	Methods	Rise time	Overshoot	Settling time
PID	Position	2.14	0.2	8.25
	Angle	2.19	0.46	9.41
FLC	Position	5.66	0	7.27
	Angle	0.45	0	16
ANFIS	Position	3.09	0	4

Angle	0.82	0	8.91

From comparison among two acumen methods for tuning a PID Controller can be concluded that

- The rise time is less in conventional PID Controller than the acumen methods.
- Settling time and overshoot has given best performance in ANFIS and PID.
- Hence, the acumen methods give the better performance when compared with conventional PID Controller.

D. Simulation of Inverted Pendulum Control Strategies

The experimental results demonstrate the effectiveness of fractional-order controllers in stabilizing an inverted pendulum system under varying conditions. The rotary arm angle and pendulum motion remain stable, highlighting the controller's ability to manage system dynamics efficiently. As shown in Figure 4, the top and middle subplots represent the rotary arm angle and pendulum motion, respectively, exhibiting smooth oscillations and stability within the desired range.

The bottom subplot in Figure 4 illustrates the control voltage, showcasing the controller’s prompt and effective response to disturbances while maintaining system stability. Fractional-order controllers outperform conventional controllers by providing precise tuning capabilities and enhanced adaptability to external uncertainties. These results confirm that fractional-order controllers significantly reduce overshoot and steady-state error, while improving robustness and response time. The findings validate the superior performance of fractional-order controllers in achieving stable and reliable control for complex dynamic systems such as the inverted pendulum.

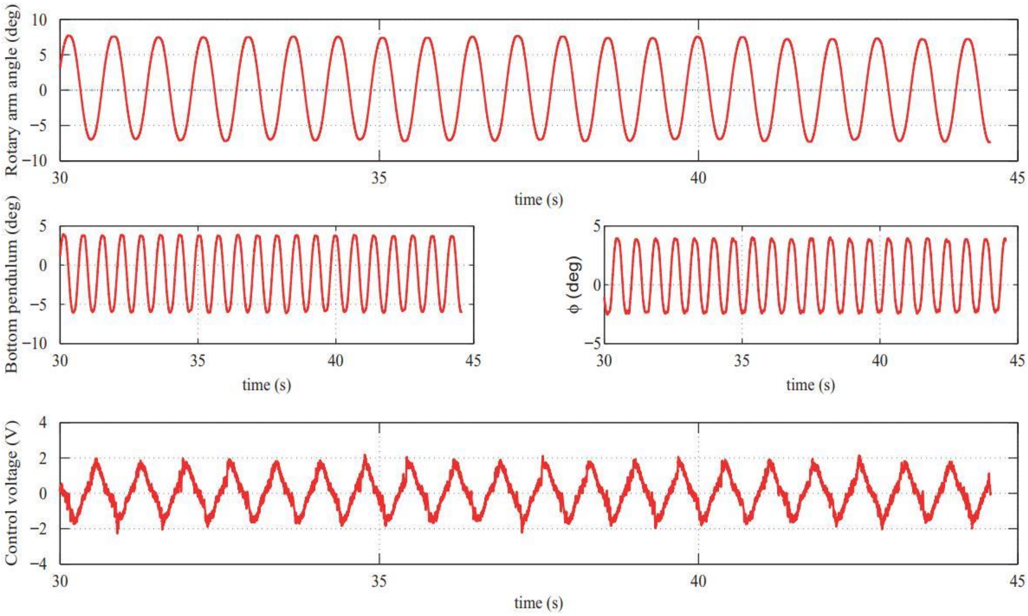

Figure 6 Experimental results for controlling an inverted pendulum system using fractional order controllers, showing stable rotary arm angle and pendulum motion. Top and middle subplots represent smooth oscillations and stability within the desired range.

E. Analysis of Integer-Order Controllers for Inverted Pendulum System Stabilization

The simulation results for controlling an inverted pendulum system using integer-order controllers. Key parameters, including the rotary arm angle, bottom pendulum angle, and top pendulum angle, are plotted over time to evaluate system stability and performance under varying control strategies. Integer-order controllers, such as PID and LQR, demonstrate effective balance control and minimized oscillations within the system.

The control voltage plot illustrates the actuator effort required to maintain stability. The LQR controller optimizes the cost function, reducing actuator effort and ensuring efficient system operation. While PID controllers exhibit robust performance in minimizing overshoot, rise time, and steady-state error, they demonstrate lower adaptability to nonlinearities and external disturbances compared to advanced control methods (Figure 5). MATLAB/Simulink simulations validate the reliability of integer-order controllers in achieving precise trajectory tracking and maintaining system stability for the inverted pendulum system.

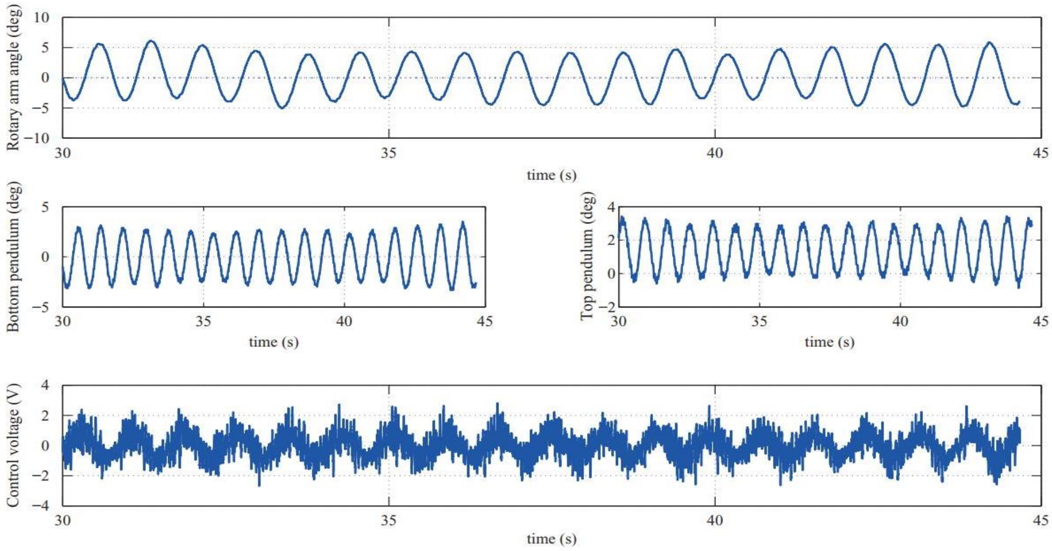

Figure 7 Comparison of LQR and PID controllers, highlighting LQR's optimization of actuator effort and PID's robust performance in reducing overshoot, rise time, and steady-state error

F. *Disturbance Rejection Performance of Various Controllers for Inverted Pendulum System*

The disturbance rejection performance of the inverted pendulum system is evaluated, focusing on key parameters such as the rotary arm angle, bottom pendulum angle, and top pendulum angle. The results highlight the system's response to external disturbances and the effectiveness of various controllers in maintaining stability (Figure 6).

Classical controllers, such as PID and LQR, effectively reduce oscillations and achieve stable control but exhibit limited robustness under significant disturbances. The LQR controller optimizes the cost function, minimizing control effort while maintaining precise trajectory tracking. Conversely, intelligent controllers, including Neural Networks (NN) and Fuzzy Logic Controllers (FLC), demonstrate superior disturbance rejection, with enhanced adaptability and robustness to system nonlinearities (Figure 6). These advanced controllers achieve improved stability, reduced oscillations, and faster recovery from disturbances, as evidenced by smoother system responses and lower control voltage variations.

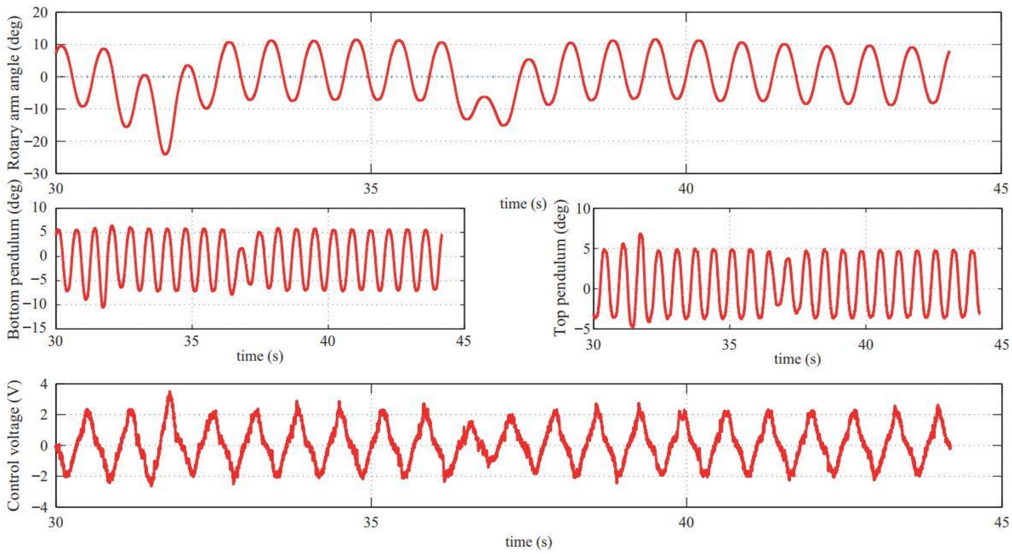

Figure 8 System performance under disturbance rejection, comparing classical controllers (PID and LQR) and intelligent controllers (NN and FLC). Intelligent controllers exhibit superior adaptability, reduced oscillations, and faster recovery, maintaining stability with lower control effort.

G. *Dynamic Response of Inverted Pendulum System to Square Reference Signal*

The experimental results for the inverted pendulum system using a square reference signal, demonstrating the dynamic response of key parameters, including the rotary arm angle (θ), bottom pendulum angle (α), top pendulum angle (ϕ), and the corresponding control voltage (Vm) (Figure 7). The rotary arm angle (top graph) closely follows the square reference signal, with oscillations observed during transitions, reflecting the controller's effort to track abrupt changes in the reference trajectory. The bottom pendulum angle (lower left) and top pendulum angle (lower right) display periodic oscillations that align with the square input, indicating the system's controlled response under the applied control strategy. The control voltage graph (bottom) highlights the actuator effort required to track the square reference signal, showing periodic control variations that correspond to the rapid trajectory changes. These results confirm the controller's effectiveness in ensuring system stability and achieving accurate tracking of the square reference signal, despite nonlinearities and external disturbances (Figure 7). The system's robust performance and minimized oscillations validate the controller's capability to maintain balance and dynamic trajectory tracking under challenging input conditions.

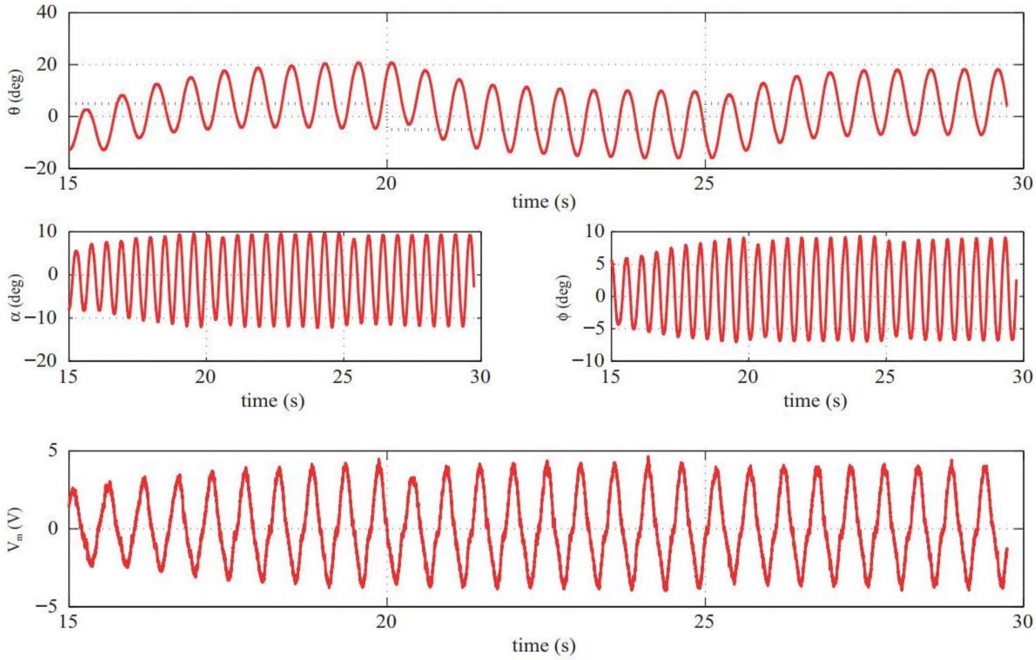

Figure 9 Experimental results for the square reference signal in the inverted pendulum system, showing the rotary arm, bottom, and top pendulum angles tracking the reference input. The control voltage reflects the actuator's efforts to stabilize the system, demonstrating effective disturbance rejection and stability. The results validate the controller's ability to maintain balance and achieve dynamic trajectory tracking.

IV. DISCUSSION

The inverted pendulum system benefits from the application of intelligent control techniques, providing solutions to complex control system problems. The primary advantage lies in the integration of adaptive algorithms, which offer more logical and efficient control compared to traditional systems [37]. The system's performance and efficiency were thoroughly assessed through simulations, highlighting the importance of implementation factors in inverted pendulum control. Fuzzy controllers (FC), neural networks (NN), and genetic algorithms are key components, with their interactions, particularly with the genetic controller, playing a crucial role in system performance [38]. This study not only demonstrates the effectiveness of these intelligent control techniques but also provides a comprehensive training environment for data handling. However, it also presents challenges that are difficult to address with the current tools and methodologies available.

The increasing availability of fuzzy logic controllers and neural network research projects highlights the significant potential for the development and real-time implementation of intelligent control methods in control systems and aerospace engineering [39]. Current efforts focus on understanding and designing real-time optimization controllers that do not rely on complete system knowledge and are adaptable to changing conditions. Flexible and optimal intelligent controller methodologies have emerged, which can be applied to any plant, regardless of its dynamics [9]. It is now possible to develop control systems without needing a detailed mathematical model of the system.

In this work, PID control and Linear Quadratic Regulator (LQR), an optimal control method, were employed to control the nonlinear inverted pendulum-cart system, both with and without continuous disturbance inputs. The results of the proposed

PID+LQR control approach were compared with those from the standalone PID control system. In the optimal control of the nonlinear inverted pendulum system, the PID controller and LQR approach account for all instantaneous states of the nonlinear system, which are assumed to be measurable and are directly input into the LQR. The LQR is developed using the linear state-space model of the system. The optimal control value from the LQR is then added negatively to the PID control value to achieve the overall optimal control input.

MATLAB-Simulink models were developed to simulate the control scheme and analyze performance [40]. The PID controllers used both as standalone PID control techniques and in combination with the PID+LQR control method, were tuned via trial and error until the desired outcomes were achieved. The simulation results validate the comparative advantage of the optimal control approach when employing the LQR technique. Despite frequent disturbances, such as wind forces, the pendulum remains stabilized in the upright position, and the cart smoothly progresses to the desired location. These results highlight the robustness and efficacy of the control systems.

Analysis of the control scheme responses demonstrates that the proposed PID+LQR control strategy outperforms the traditional PID control [41]. This strategy provides a straightforward, efficient, and reliable approach to the optimal control of nonlinear dynamic systems, as evidenced by the comparative performance analysis of the benchmark system. Future research could further investigate the effectiveness of this control strategy by tuning the PID controller parameters using Genetic Algorithms (GA) and Particle Swarm Optimization (PSO), as opposed to the trial-and-error method currently employed.

The inverted pendulum system consists of a suspended weight capable of rotating around its pivot point and is widely studied in control systems [42]. It serves as a classic example of a nonlinear system and has been extensively used to evaluate various control algorithms [43]. This paper aims to compare three of the most commonly used control algorithms for stabilizing the inverted pendulum: Proportional-Integral-Derivative (PID) Control, Linear Quadratic Regulator (LQR) Control, and Fuzzy Logic Control.

Proportional-Integral-Derivative (PID) Control is one of the most widely used control systems for managing the inverted pendulum [44]. It is a feedback control strategy that employs proportional, integral, and derivative components to monitor and adjust the system's output. The PID controller utilizes a mathematical model of the system to determine the required adjustments to the output [45]. While this controller is relatively simple to implement and can yield satisfactory results in certain cases, it is often challenging to tune. Additionally, the PID controller may exhibit slow response times when reacting to changes in the system, which can hinder its effectiveness in dynamic scenarios.

Linear Quadratic Regulator (LQR) Control is another widely used control strategy for stabilizing the inverted pendulum [46]. The LQR controller uses a state-space representation of the system and a cost function to optimize performance by minimizing a weighted sum of the system's state variables and control inputs [47]. While the implementation of the LQR controller is more complex compared to the PID controller, it offers enhanced reliability and can deliver superior performance. Notably, the LQR controller is able to respond more rapidly to system changes, providing faster stabilization and better handling of dynamic variations.

The final control strategy discussed in this paper is Fuzzy Logic Control (FLC). Fuzzy logic is a reasoning approach based on approximate rather than precise reasoning. It utilizes a set of fuzzy rules to make decisions and adjust the system's output accordingly. The fuzzy logic controller does not require a detailed mathematical model of the system, making it easier to implement in certain cases. While FLC can produce satisfactory results in many instances, it can be challenging to tune due to the dependence on heuristic rules. Additionally, it may exhibit slower response times when adapting to changes in the system, which can limit its effectiveness in highly dynamic environments.

When comparing the performance of PID, LQR, and Fuzzy Logic Control systems for regulating an inverted pendulum, several key metrics are essential for assessing system success. The most critical metrics include stability, response speed, and output accuracy [48].

Regarding stability, the PID controller is generally the most reliable among the three, it achieves good stability performance and is relatively easy to tune [49]. The LQR controller also demonstrates strong stability, with performance exceeding that of the PID controller, particularly in more complex scenarios. The fuzzy logic controller, while effective in certain cases, is less reliable than the PID and LQR controllers in maintaining stability under varying conditions [50]. In terms of response speed, the PID controller typically offers the fastest response. The LQR controller also provides strong performance in this area, though it is slightly slower than the PID controller. The fuzzy logic controller, however, tends to have a slower response time compared to both PID and LQR, though it can still yield satisfactory results under certain conditions. Finally, when evaluating accuracy, the PID controller is generally the most accurate. The LQR controller also provides good accuracy, although it is marginally less precise than the PID controller. The fuzzy logic controller, while capable of delivering reasonable results in specific applications, generally lags behind the other two controllers in terms of accuracy. However, it may still offer adequate performance in some cases.

In conclusion, the Proportional-Integral-Derivative (PID), Linear Quadratic Regulator (LQR) and Fuzzy Logic Control systems are all viable options for controlling an inverted pendulum. The PID controller is the most reliable and accurate of the three control systems, while the LQR controller is slightly less reliable but more responsive. The fuzzy logic controller is not as reliable as the other two controllers, but it can provide good results in some cases. Each of the three control systems has its own strengths and weaknesses, and it is important to consider all of the factors when deciding which one is best suited for the task at hand.

Conclusion

This study explores the application of intelligent control techniques, including Genetic Algorithms, ANFIS (Adaptive Neuro-Fuzzy Inference Systems), Fuzzy Logic, and Neural Networks, for controlling the inverted pendulum system. Genetic algorithms were proposed for integrating various control methods, while the combination of Genetic-ANFIS controllers and Neural Fuzzy (NF) approaches leveraged advancements in intelligent control strategies. A comprehensive strategy for determining the optimal controller design was presented.

Linear Quadratic Regulator (LQR), a classical linear control technique, demonstrated effective system stabilization with faster settling times compared to the PID controller. The PID controller, through manual tuning, minimized steady-state error and effectively managed overshoot within the desired range. In contrast, Fuzzy Logic Controllers, utilizing the Mamdani method, offered model-free control, successfully stabilizing the pendulum angle without the need for a mathematical model. Neural Networks were implemented to reduce external disturbances and uncertainties. By introducing external disturbances in the control input, the system remained stable, with effective training for weight adjustment. The study demonstrated that ANFIS outperforms both PID and Fuzzy Logic Controllers in terms of control performance. Future work could focus on using advanced neuro-fuzzy architectures, such as FALCON, GARIC, or NEFCON, for further improvements in inverted pendulum control.

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