



Diagonally Loaded LMS Beamforming for Underwater Acoustic Arrays: Steady-State Limits and Robustness in Anisotropic, Impulsive, and Non-Stationary Noise

Sehrish^{1*}, Athar Rasool², Dr. Syed Irfan Hussain³, Hanan Ahmad Saeed⁴

^{1,3}University of Engineering & Technology Lahore

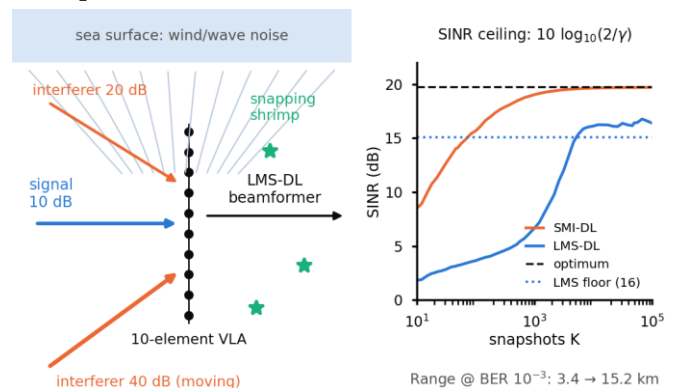
^{2,4}Department of Nuclear Engineering, Pakistan Institute of Engineering and Applied Sciences (PIEAS), Islamabad, Pakistan
 sehrish.qamar27@gmail.com¹, ather.rasool@gmail.com², ssirfanhussain@uet.edu.pk³, hana_a_saeed@hotmail.com⁴

*Corresponding author: Sehrish (sehrish.qamar27@gmail.com)

Abstract—Adaptive array processing is a key enabler of interference-resilient underwater acoustic (UWA) links, yet the noise seen by UWA receivers is spatially anisotropic, impulsive, and non-stationary, conditions under which low-complexity stochastic-gradient beamformers are poorly characterized. This paper analyzes the Griffiths-type least-mean-squares beamformer with diagonal loading and a power-normalized step size (LMS-DL) on a ten-element half-wavelength line array, and benchmarks it against sample-matrix-inversion (SMI-DL), normalized LMS (NLMS-DL), and exponentially weighted recursive-least-squares (EW-RLS-DL) beamformers. A closed-form approximation of the steady-state output signal-to-interference-plus-noise ratio (SINR) of LMS-DL is derived. It shows that the misadjustment is driven by the total output power, so the SINR saturates at $10\log_{10}(2/\gamma)$ dB for step-size parameter γ ; Monte Carlo simulations show the prediction is conservative and accurate to within 0.75 dB at $\text{SNR} \leq 10$ dB. For a 10 dB signal and interferers at 20 and 40 dB, LMS-DL settles 3.3 dB below the optimum after about 10^4 snapshots, whereas SMI-DL comes within 1 dB in 681 snapshots. In Cron-Sherman surface-generated noise the optimum gains 7.4 dB over white noise; SMI-DL recovers 6.4 dB of this gain but LMS-DL only 2.5 dB. Symmetric α -stable noise with $\alpha = 1.2$ costs LMS-DL 5.6 dB, a loss that per-snapshot normalization (NLMS-DL) reduces to 0.3 dB. LMS-DL tracks a moving 40 dB interferer (worst-case loss 4.1 dB) but not a moving 20 dB interferer (13.3 dB). A link budget with Thorp absorption and Wenz noise shows that converged LMS-DL extends the QPSK range at a bit-error rate (BER) of 10^{-3} from 3.4 km to 15.2 km (optimum 16.0 km), but leaves an error floor of $2-5 \times 10^{-5}$ at short range. The analysis yields design rules for the step size and for SINR-dependent loading in UWA array receivers.

Keywords— Underwater acoustic communications, adaptive beamforming, LMS algorithm, diagonal loading, misadjustment, ambient-noise coherence, symmetric α -stable noise, interference suppression, underwater acoustic networks.

Graphical Abstract



I. INTRODUCTION

Underwater acoustic (UWA) networks support environmental monitoring, offshore infrastructure, and fleets of autonomous vehicles [1]–[3]. Their physical layer operates in one of the most difficult channels in communications. The usable bandwidth is a few kilohertz and shrinks with range because absorption grows rapidly with frequency [4]. Propagation is dominated by time-varying multipath and Doppler [5]–[8]. Recent surveys review these challenges and the open problems of networked operation [9], [10].

Two features of the underwater noise environment make receiver design particularly demanding. First, ambient noise generated by wind, waves, shipping, and turbulence is not spatially white: it arrives preferentially from the sea surface and from distant shipping lanes, so its spatial coherence across an array depends on geometry and frequency [11]–[14]. Second, in warm shallow waters snapping shrimp produce broadband impulsive noise that is well described by symmetric α -stable (SaS) statistics, with heavy tails that break detectors and receivers designed for Gaussian noise [15], [16]. In a network, co-channel transmissions from other nodes and noise from nearby vessels add strong directional interferers, which may also move.

Spatial processing with hydrophone arrays has long been used to counter these effects. Multichannel combining and phase-coherent receivers exploit spatial diversity [17]–[19], and multicarrier receivers compensate wideband Doppler distortion [20], [21]. More recently, deconvolved beamforming has enabled long-range deep-sea links [22], and transmit beamforming toward the dominant propagation path has been demonstrated with at-sea data [23]. A recent review summarizes the state of underwater beamforming and highlights robustness to non-stationary noise and model mismatch as a key challenge [24].

Adaptive beamforming theory provides the tools for interference suppression. The minimum-variance distortionless-response (MVDR) beamformer [25] and its linearly constrained generalization [26] maximize the output SINR when the interference-plus-noise covariance is known [27]–[29]. In practice the weights are learned from data, either by sample-matrix inversion (SMI), whose convergence is governed by the Reed–Mallett–Brennan rule [30], or by adaptive-array algorithms such as the LMS array [31] and the Applebaum array [32]. The Griffiths variant replaces the reference signal with the known look-direction cross-correlation vector [33]. The convergence and misadjustment of LMS-type arrays have been analyzed extensively for Gaussian data [34]–[37]. Normalized LMS [38], variable step-size schemes [39], and recursive least squares (RLS) [40] trade complexity against convergence and tracking speed.

Finite-sample effects and model mismatch are handled by robust adaptive beamforming. Mismatch between the presumed and actual signal signature causes signal self-cancellation [41], [42]. Diagonal loading is the classical remedy [43], [44], and robust Capon and worst-case designs reduce to (adaptive) diagonal loading under ellipsoidal or spherical steering uncertainty [45], [46]; see [47] for a unified treatment and [48] for covariance-reconstruction approaches. Sonar-specific adaptive processors include dominant-mode rejection [49], deconvolved conventional beamforming [50], transmission-diversity smoothing for MIMO sonar [51], joint matched-filter/beamformer designs [52], high-resolution processing of combined waveforms [53], main-lobe interference suppression for small arrays [54], and adaptive focusing for wideband beamforming in multipath [55]. For impulsive noise, fractional lower-order moments [56], [57], covariation-based subspace methods [58], spatial-sign statistics [59], and M-estimators of scatter [60], [61] replace the Gaussian sample covariance.

LMS-type beamformers remain attractive for energy-constrained underwater nodes because their cost grows only linearly with the number of sensors. To the best of our knowledge, however, their behavior has been characterized mainly for spatially white Gaussian noise and static interference. A joint, quantitative treatment of the diagonally loaded Griffiths LMS under anisotropic ambient noise, impulsive noise, moving interferers, and a physical link budget has not been reported. This paper provides such a treatment. Its contributions are as follows.

1) **Steady-state analysis.** A closed-form approximation of the steady-state SINR of LMS-DL is derived (16, 17). It shows that the misadjustment is driven by the total output power,

including the desired signal, so the SINR saturates at $10\log_{10}(2/\gamma)$ dB as the SNR grows. Monte Carlo simulation confirms the prediction to within 0.75 dB for $\text{SNR} \leq 10$ dB, and it remains conservative up to 45 dB.

- 2) **Convergence mechanisms.** The learning curve and beampattern evolution are explained by the eigenmode time constants of the loaded covariance, and compared with SMI-DL, NLMS-DL, and EW-RLS-DL under identical conditions.
- 3) **Underwater noise fields.** Performance is quantified for 3-D and 2-D isotropic noise, Cron–Sherman surface-generated noise, temporally colored $1/f$ noise, and S&S impulsive noise.
- 4) **Design rules.** The results give a step-size rule from a target SINR, SNR-dependent loading under array mismatch, per-snapshot normalization for impulsive noise, and RLS for weak moving interferers.
- 5) **Link-level evaluation.** BER is related to range through Thorp absorption and Wenz ambient noise, which exposes an LMS-specific error floor at short range.

The remainder of the paper is organized as follows. Section II presents the signal and noise models. Section III describes the beamformers and derives the steady-state floor. Section IV gives the simulation setup. Section V presents the results, Section VI discusses implications and limitations, and Section VII concludes.

II. SYSTEM MODEL

A. Array and Signal Model

Consider a uniform linear array (ULA) of N omnidirectional hydrophones at positions $z_n = (n - N - 1/2)d$, $n = 0, \dots, N - 1$, with spacing $d = \lambda/2$ at the carrier wavelength λ . For a plane wave arriving with direction cosine u relative to the array axis (so that $u = 0$ is broadside), the steering vector is

$$\mathbf{v}(u) = [e^{j2\pi uz_0/\lambda}, \dots, e^{j2\pi uz_{N-1}/\lambda}]^T. \quad (1)$$

Under the narrowband assumption [29], the snapshot received at time k is

$$\mathbf{x}(k) = \mathbf{v}_s s(k) + \sum_{i=1}^2 \mathbf{v}_{i,i} I_i(k) + \mathbf{n}(k), \quad (2)$$

where $\mathbf{v}_s = \mathbf{v}(u_s)$ and $\mathbf{v}_{i,i} = \mathbf{v}(u_{i,i})$. The signal $s(k)$ and the interferers $I_i(k)$ are zero-mean circular Gaussian with powers σ_s^2 and $\sigma_{i,i}^2$. The noise $\mathbf{n}(k)$ has covariance $\sigma_w^2 \mathbf{Q}_n$ with $\text{tr}(\mathbf{Q}_n) = N$. The interference-plus-noise and input covariance matrices are

$$\mathbf{S}_n = \sum_{i=1}^2 \sigma_{i,i}^2 \mathbf{v}_{i,i} \mathbf{v}_{i,i}^H + \sigma_w^2 \mathbf{Q}_n, \quad (3)$$

$$\mathbf{S}_x = \mathbf{S}_n + \sigma_s^2 \mathbf{v}_s \mathbf{v}_s^H. \quad (4)$$

For a weight vector $\mathbf{w}$ with output $y(k) = \mathbf{w}^H \mathbf{x}(k)$, the output SINR and its maximum, attained by the MVDR beamformer $\mathbf{w} \propto \mathbf{S}_n^{-1} \mathbf{v}_s$ [25], [29], are

$$\text{SINR}(\mathbf{w}) = \frac{\sigma_s^2 |\mathbf{w}^H \mathbf{v}_s|^2}{\mathbf{w}^H \mathbf{S}_n \mathbf{w}}, \quad (5)$$

$$\text{SINR}_{\text{opt}} = \sigma_s^2 \mathbf{v}_s^H \mathbf{S}_n^{-1} \mathbf{v}_s. \quad (6)$$

The beampattern normalized to the look direction is

$$B(u) = \frac{|\mathbf{w}^H \mathbf{v}(u)|^2}{|\mathbf{w}^H \mathbf{v}_s|^2}. \quad (7)$$

B. Ambient-Noise Fields

A spatially extended noise field with directional density $f(u)$ over direction cosine produces the normalized covariance [13], [14]

$$\mathbf{Q}_n = (1 - \epsilon) \frac{N \Phi}{\text{tr}(\Phi)} + \epsilon \mathbf{I}, \quad (8)$$

where $\Phi = \int_{-1}^1 f(u) \mathbf{v}(u) \mathbf{v}^H(u) du$ and $\epsilon = 0.05$ models uncorrelated sensor self-noise. Four fields are considered: (i) white noise, $\mathbf{Q}_n = \mathbf{I}$; (ii) 3-D isotropic noise, $f(u) = 1$, whose coherence between elements m apart is $\text{sinc}(2md/\lambda)$; (iii) 2-D (cylindrical) isotropic noise, $f(u) = (1 - u^2)^{-1/2}$, with coherence $J_0(2\pi md/\lambda)$ [13]; and (iv) surface-generated noise on a vertical array. For the last, the Cron-Sherman model with exponent $n = 1$ [13] is used, with $u > 0$ pointing toward the surface and a bottom-reflected component attenuated by a power reflection coefficient of 0.3:

$$f(u) = \begin{cases} u, & u > 0, \\ 0.3|u|, & u < 0. \end{cases} \quad (9)$$

C. Impulsive Noise

Snapping-shrimp noise is modeled as SαS with characteristic exponent $\alpha \in (1, 2]$ [15], [16], [57]. We use the sub-Gaussian vector model $\mathbf{n}(k) = \sqrt{A(k)} \mathbf{g}(k)$, with $\mathbf{g}(k) \sim \mathcal{CN}(\mathbf{0}, \sigma_w^2 \mathbf{I})$ and $A(k)$ a totally skewed positive stable variable of index $a = \alpha/2$. The latter is generated with Kanter's representation [62] (see also [63]):

$$A = \frac{\sin(aU)}{(\sin U)^{1/a}} \left(\frac{\sin((1-a)U)}{W} \right)^{(1-a)/a}, \quad (10)$$

where $U \sim \mathcal{U}(0, \pi)$ and $W \sim \text{Exp}(1)$. For $\alpha = 2$, $A \equiv 1$ and the model reduces to Gaussian noise with the same dispersion. Because second moments are infinite for $\alpha < 2$, the output SINR is not defined. Each learned weight vector is therefore scored under the nominal Gaussian $\mathbf{S}_n$, which isolates the damage that impulses cause to the adaptation itself.

D. Temporally Colored Noise

Temporally colored ambient noise is generated per element by spectral shaping of complex white Gaussian noise $G(f)$, $\mathbf{n}(k) = \mathcal{F}^{-1}\{G(f) |f|^{-\kappa/2}\}$, with $\kappa = 1$ (1/f, or “pink”, noise) and normalized to unit power. The spatial covariance remains $\mathbf{Q}_n = \mathbf{I}$.

III. ADAPTIVE BEAMFORMERS

A. Griffiths LMS With Diagonal Loading (LMS-DL)

The Griffiths algorithm [33] replaces the reference signal of the LMS array [31] with the cross-correlation vector $\mathbf{p} = E\{\mathbf{x}(k)s^*(k)\} = \sigma_s^2 \mathbf{v}_s$, which is known once the look direction and the signal power are known. Diagonal loading of level $\sigma_L^2 = \text{LNR} \cdot \sigma_w^2$ [43], [44] and a power-normalized step size $\mu_k = \gamma/\hat{\sigma}_x^2(k)$, with the recursive power estimate

$$\hat{\sigma}_x^2(k) = \beta \hat{\sigma}_x^2(k-1) + (1 - \beta) \|\mathbf{x}(k)\|^2, \quad (11)$$

give the update

$\mathbf{w}(k+1) = (1 - \mu_k \sigma_L^2) \mathbf{w}(k) + \mu_k [\mathbf{p} - \mathbf{x}(k)y^*(k)]$, (12) with $\mathbf{w}(0) = \mathbf{v}_s/N$ (the conventional beamformer) and $\hat{\sigma}_x^2(0) = \gamma/\mu_{\text{SD}}$. Taking expectations shows that the mean weight converges to the Wiener point

$$\mathbf{w}_\infty = (\mathbf{S}_x + \sigma_L^2 \mathbf{I})^{-1} \mathbf{p}, \quad (13)$$

which is proportional to the diagonally loaded MPDR weight. Since σ_L^2 is small compared with the interference eigenvalues, $\text{SINR}(\mathbf{w}_\infty)$ is within 0.02 dB of SINR_{opt} in the baseline scenario of Section IV.

B. Steady-State SINR Floor

Let $\bar{\mu} = \gamma/\text{tr}(\mathbf{S}_x)$ denote the average step size and $\boldsymbol{\epsilon}(k) = \mathbf{w}(k) - \mathbf{w}_\infty$ the weight error. Unlike the error-driven LMS, whose excess mean-square error scales with the minimum MSE [36], [37], the Griffiths update is driven by the gradient noise $\mathbf{g}(k) = (\mathbf{x}(k)\mathbf{x}^H(k) - \mathbf{S}_x)\mathbf{w}(k)$. For circular Gaussian data, Isserlis' theorem gives, near convergence,

$$E\{\mathbf{g}(k)\mathbf{g}^H(k)\} \approx P_{\text{out}} \mathbf{S}_x, \quad (14)$$

where $P_{\text{out}} = \mathbf{w}_\infty^H \mathbf{S}_x \mathbf{w}_\infty$ is the total output power, which includes the desired signal. Under the usual independence and small-step assumptions [34], [35], the weight-error covariance $\mathbf{C}_w = E\{\boldsymbol{\epsilon}\boldsymbol{\epsilon}^H\}$ satisfies the Lyapunov equation $(\mathbf{S}_x + \sigma_L^2 \mathbf{I})\mathbf{C}_w + \mathbf{C}_w(\mathbf{S}_x + \sigma_L^2 \mathbf{I}) = \bar{\mu} P_{\text{out}} \mathbf{S}_x$. Because the two matrices commute, its solution is

$$\mathbf{C}_w = \frac{\bar{\mu} P_{\text{out}}}{2} \mathbf{S}_x (\mathbf{S}_x + \sigma_L^2 \mathbf{I})^{-1}. \quad (15)$$

The weight jitter adds $\text{tr}(\mathbf{S}_n \mathbf{C}_w)$ to the output interference-plus-noise power, which gives the steady-state SINR

$$\text{SINR}_\infty \approx \frac{\sigma_s^2 |\mathbf{w}_\infty^H \mathbf{v}_s|^2}{\mathbf{w}_\infty^H \mathbf{S}_n \mathbf{w}_\infty + \text{tr}(\mathbf{S}_n \mathbf{C}_w)}. \quad (16)$$

When the loading is small relative to the interference eigenvalues, (16) reduces to

$$\text{SINR}_\infty \approx \frac{\text{SINR}_{\text{opt}}}{1 + \frac{\gamma}{2}(1 + \text{SINR}_{\text{opt}})}. \quad (17)$$

In the baseline scenario, (16) and (17) agree to within 0.01 dB. Equation (17) has two practical consequences. First, for a target steady-state SINR it fixes the largest admissible step, $\gamma \leq 2(\text{SINR}_{\text{opt}}/\text{SINR}_{\text{target}} - 1)/(1 + \text{SINR}_{\text{opt}})$. Second, since $\text{SINR}_\infty \rightarrow 2/\gamma$ as $\text{SINR}_{\text{opt}} \rightarrow \infty$, it caps the output SINR at $10\log_{10}(2/\gamma)$ dB (17 dB for $\gamma = 0.04$), regardless of transmit power. The transient is governed by the eigenvalues λ_i of $\mathbf{S}_x$ through the modal time constants [36], [37]

$$\tau_i = \frac{1}{\bar{\mu}(\lambda_i + \sigma_L^2)}, \quad (18)$$

so the eigenvalue spread created by strong interferers and anisotropic noise directly sets the convergence time.

C. Benchmark Beamformers

NLMS-DL. The recursion (12) is used with the per-snapshot step [38]

$$\mu_k = \frac{\gamma}{\delta + \|\mathbf{x}(k)\|^2}, \quad (19)$$

which bounds the effective step $\mu_k \|\mathbf{x}(k)\|^2$ by γ . Here $\gamma = 0.01$ is chosen so that NLMS-DL and LMS-DL reach the same steady state in Gaussian noise.

SMI-DL (loaded MPDR). The weights are computed from the sample covariance $\hat{\mathbf{R}} = K^{-1} \sum_{k=1}^K \mathbf{x}(k)\mathbf{x}^H(k)$ [30], [44]:

$$\mathbf{w}_{\text{SMI}} = (\hat{\mathbf{R}} + \sigma_L^2 \mathbf{I})^{-1} \mathbf{v}_s. \quad (20)$$

Because the desired signal is present in the training data, this MPDR form is more sensitive to finite K and to mismatch than MVDR, and loading is essential [29], [41]. An unloaded SMI is also shown for reference.

EW-RLS-DL. An exponentially weighted covariance tracks non-stationary interference [37], [40]:

$$\hat{\mathbf{R}}(k) = \lambda \hat{\mathbf{R}}(k-1) + (1-\lambda) \mathbf{x}(k)\mathbf{x}^H(k), \quad (21)$$

with $\mathbf{w}(k) = (\hat{\mathbf{R}}(k) + \sigma_L^2 \mathbf{I})^{-1} \mathbf{v}_s$ and forgetting factor $\lambda \in \{0.99, 0.999\}$. The constant loading term prevents an exact rank-one inverse update, so in the simulations the weights are recomputed every 100 snapshots.

Table I summarizes the computational cost of the four beamformers.

TABLE I
COMPUTATIONAL COST PER SNAPSHOT

Beamformer	Complex multiplications	Memory	Main parameter
LMS-DL	$\approx 4N$	$O(N)$	$\gamma, \beta, \text{LNR}$
NLMS-DL	$\approx 4N$	$O(N)$	$\gamma, \delta, \text{LNR}$
SMI-DL	N^2 per snapshot + $O(N^3)$ per solve	$O(N^2)$	K, LNR
EW-RLS-DL	N^2 per snapshot + $O(N^3)$ per solve	$O(N^2)$	λ, LNR

IV. SIMULATION SETUP

Table II lists the parameters. The baseline scenario places the desired signal at broadside with a per-element SNR of 10 dB, and two interferers at $u = 0.29$ (INR = 20 dB) and $u = -0.45$ (INR = 40 dB). Output SINR is averaged over Monte Carlo trials in linear scale. Because the LMS distribution is skewed, medians and 10th–90th percentiles are also reported.

TABLE II
SIMULATION PARAMETERS

Parameter	Value
Array	$N = 10$ ULA, $d = \lambda/2$ (vertical line array in Secs. V-D to V-G)
Desired signal	$u_s = 0$, SNR = 10 dB per element
Interferers	$u = 0.29$ (INR = 20 dB), $u = -0.45$ (INR = 40 dB)
LMS-DL	$\gamma = 0.04, \beta = 0.99, \mu_{\text{SD}} = 4.9995 \times 10^{-6}$, LNR = 10 dB
NLMS-DL / EW-RLS-DL	$\gamma = 0.01 / \lambda \in \{0.99, 0.999\}$, same LNR
Snapshots, trials	$K \leq 10^5$; 200 trials (Figs. 1–2), 100 (Figs. 3–6), 2000 (Fig. 7)
Noise fields	White, 3-D/2-D isotropic, Cron–Sherman ($n = 1$), $1/f$, S&S with $\alpha \in \{2, 1.8, 1.5, 1.2\}$
Link budget	$f_c = 12$ kHz, $B = 4$ kHz, SL = 170 dB re 1 μPa at 1 m, $k = 1.5$
Absorption, noise	Thorp: 1.645 dB/km at 12 kHz; Wenz (shipping 0.5, wind 5 m/s): 80.6 dB re 1 μPa in band

For the link-level study, the transmission loss and the Thorp absorption coefficient are [4], [12], [64], with α_{dB} in dB/km and f in kHz

$$\text{TL}(r) = 10k \log_{10} \frac{r}{1 \text{ m}} + \alpha_{\text{dB}}(f) \frac{r}{1 \text{ km}}, \quad (22)$$

$$\alpha_{\text{dB}}(f) = \frac{0.11f^2}{1+f^2} + \frac{44f^2}{4100+f^2} + 2.75 \times 10^{-4} f^2 + 0.003. \quad (23)$$

The ambient noise level follows the empirical Wenz turbulence, shipping, wind, and thermal components [4], [11]. The per-element SNR is $\text{SL} - \text{TL}(r) - \text{NL}$, which equals 42.7, 25.7, 12.9, 2.0, and -8.0 dB at 1, 5, 10, 15, and 20 km. The residual interference-plus-noise at the beamformer output is Gaussian, so for QPSK [65]

$$P_b = E \left\{ Q \left(\sqrt{\text{SINR}(\mathbf{w})} \right) \right\}, \quad (24)$$

where the expectation is taken over the random weight realizations. The interferer INRs are held fixed as the range of the desired node varies.

V. RESULTS AND DISCUSSION

A. Convergence and Steady-State SINR

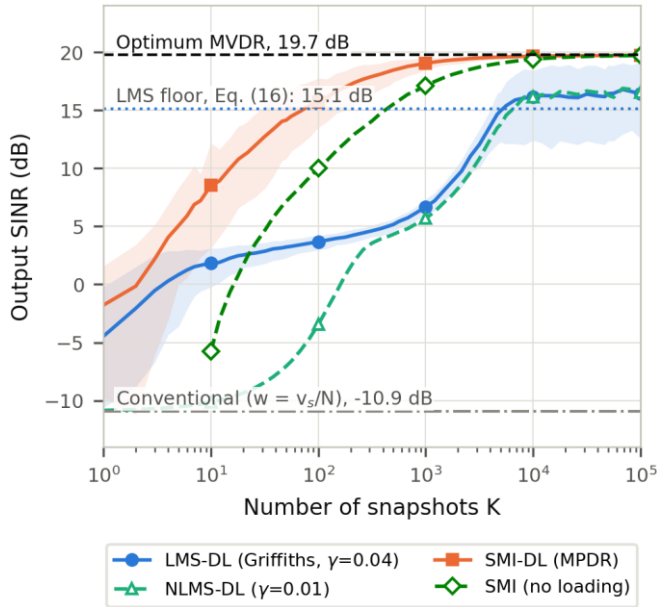


Figure 1. Output SINR versus number of snapshots ($N = 10$, $\text{SNR} = 10$ dB, $\text{INR} = 20/40$ dB, $\text{LNR} = 10$ dB; 200 trials). Shaded bands are 10th–90th percentiles; the dotted line is the floor predicted by (16).

Fig. 1 and Table III show that LMS-DL settles at 16.4 dB after about 10^4 snapshots, 3.3 dB below the optimum of 19.74 dB. SMI-DL comes within 1 dB of the optimum after 681 snapshots. The conventional beamformer reaches only -10.9 dB because its sidelobe passes the 40 dB interferer. The learning curve of LMS-DL has three phases that match the modal time constants (18) with $\bar{\mu} = 3.96 \times 10^{-7}$. The 40 dB interferer mode ($\lambda \approx 10^5$, $\tau \approx 25$) is nulled almost immediately. The 20 dB interferer mode ($\lambda \approx 10^3$, $\tau \approx 2.5 \times 10^3$) produces the slow climb between $K = 10^2$ and 5×10^3 . The signal-subspace mode ($\lambda \approx 95$, $\tau \approx 2.4 \times 10^4$) drifts slowly, but the output SINR is by then limited by gradient noise.

The predicted floor of 15.1 dB from (16) lies 0.5–1 dB below the simulated median (15.6–16.1 dB), so the approximation is conservative. The Wiener point itself achieves 19.72 dB, so the whole 3.3 dB gap is misadjustment, not loading bias. LMS-DL also remains widely spread across trials (P10–P90 of 12.6–19.1 dB at $K = 10^5$), whereas SMI-DL concentrates within 19.69–19.72 dB. The penalty of unloaded SMI reflects the known sensitivity of MPDR training when the signal is present [30], [41], which 10 dB of loading largely removes [44].

TABLE III
OUTPUT SINR (dB) VERSUS SNAPSHOTS (MEAN OF 200 TRIALS; OPTIMUM 19.74 dB)

K	LMS-DL	NLMS-DL	SMI-DL	SMI
10	1.83	-10.14	8.51	-5.78
100	3.65	-3.46	15.68	9.99
1,000	6.66	5.69	19.03	17.08
5,000	14.95	13.78	19.56	19.04
10,000	16.12	16.11	19.64	19.38
100,000	16.40	16.52	19.71	19.70
K to optimum – 6 dB	4,642	$\approx 5,000$	46	316
K to optimum – 1 dB	not reached	not reached	681	3,831

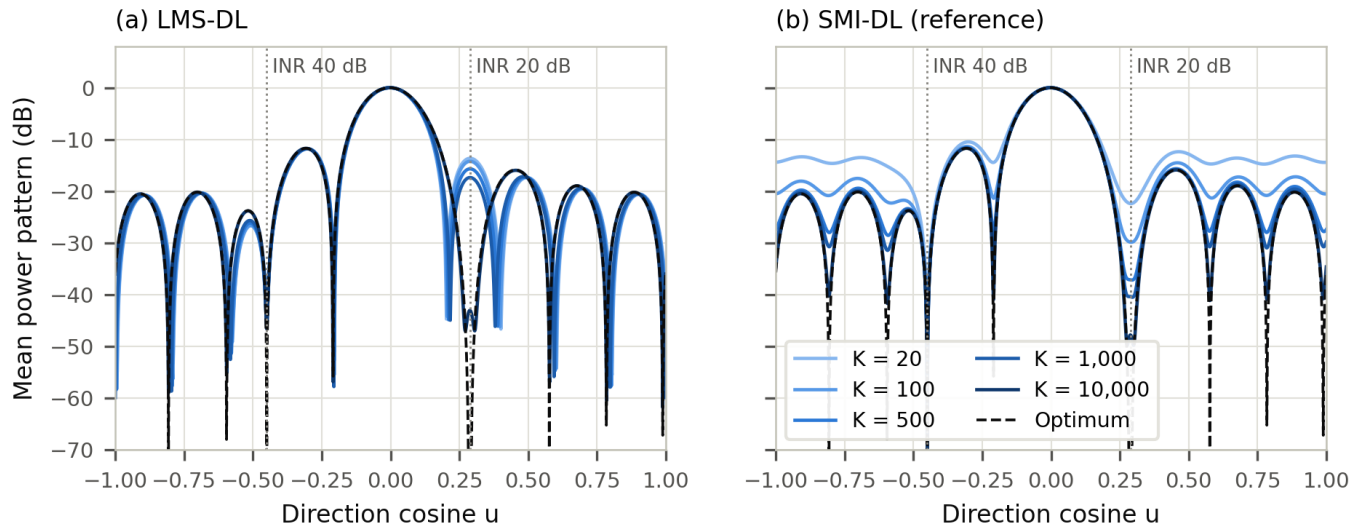


Figure 2. Mean power patterns normalized to the look direction at $K \in \{20, 100, 500, 1000, 10000\}$ (200 trials): (a) LMS-DL; (b) SMI-DL. Dotted verticals mark the interferers.

B. Beam pattern Evolution

Fig. 2 shows that LMS-DL nulls the 40 dB interferer within 20 snapshots but leaves the 20 dB interferer on the conventional sidelobe (-13.6 to -17.4 dB) until $K = 10^3$. The null on that interferer forms only by $K = 10^4$ (-43.1 dB). The null on the 40 dB interferer stops deepening at about -46.6 dB, whereas the optimum places nulls of -72.8 dB and -120.4 dB. This is the misadjustment of (16) seen in the pattern: with unit look-

direction gain, a -46.6 dB null leaves residual interference of $0.22\sigma_w^2$, roughly twice the optimum output noise of $0.11\sigma_w^2$. SMI-DL nulls both interferers by $K = 20$ (-22.4 and -42.5 dB), at the cost of far sidelobes raised by 5.7 dB (peak) and 8.8 dB (average) for $|u| \geq 0.6$. By $K = 500$ this excess falls to 0.7 and 1.3 dB, the finite-sample effect that diagonal loading is designed to contain [43], [44].

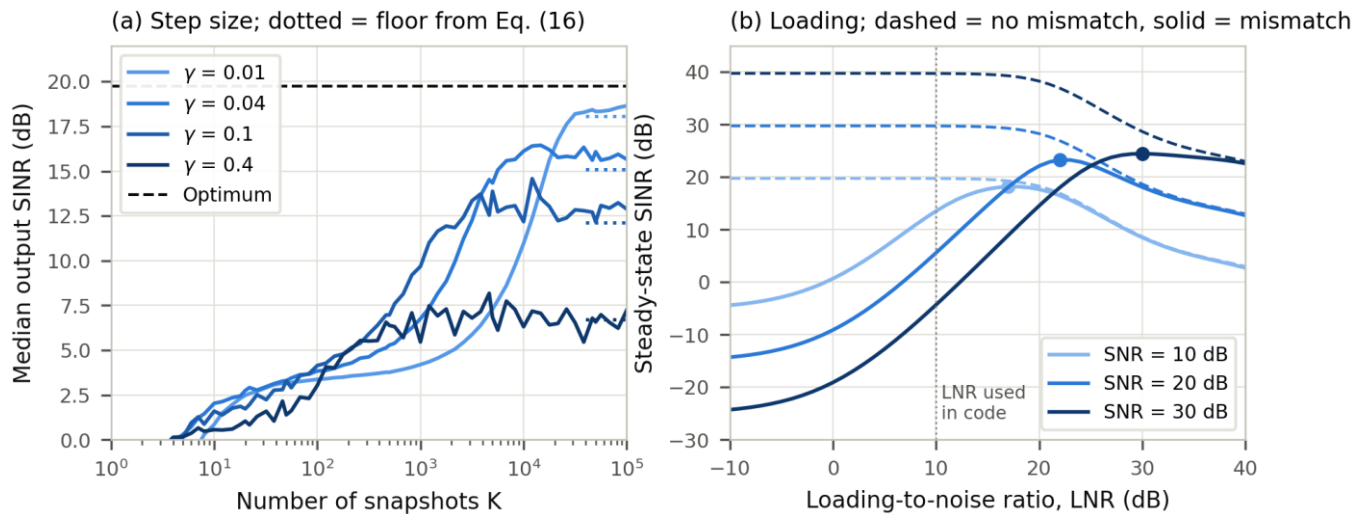


Figure 3. (a) Median SINR of LMS-DL for $\gamma \in \{0.01, 0.04, 0.1, 0.4\}$ (100 trials); dotted segments are the floors from (16). (b) Steady-state SINR of w_∞ versus LNR without (dashed) and with (solid) mismatch: element-position errors of standard deviation 0.05λ plus a 0.02 look-direction error, averaged over 500 array realizations.

C. Step Size and Diagonal Loading

For every step size in Fig. 3(a), (16) predicts the median floor to within 0.75 dB, always on the conservative side. The predictions are 18.05, 15.11, 12.14, and 6.72 dB for $\gamma = 0.01, 0.04, 0.1$, and 0.4 , and the simulated medians at $K = 10^5$ are 18.65, 15.66, 12.88, and 7.23 dB. Speed and floor trade off directly: at $K = 10^3$ the median SINR is 9.7 dB for $\gamma = 0.1$ but 4.2 dB for $\gamma = 0.01$, while the final values are reversed by about

6 dB. A fixed step therefore cannot be both fast and accurate, which motivates variable step-size schemes [39].

Fig. 3(b) addresses loading. Without mismatch, the loss stays below 0.2 dB up to LNR = 15 dB and reaches 1.5 dB at 20 dB. With array perturbations, the signal leaks into the adaptive nulling and self-cancels [41], [42]. The best LNR is then 17, 22, and 30 dB for SNR = 10, 20, and 30 dB, giving 18.18, 23.27, and 24.45 dB. The fixed 10 dB loading yields only 13.54, 5.60, and -4.29 dB, a loss of 17.7 dB at 20 dB SNR. Above 20 dB

SNR the best LNR lies 0–2 dB above the SNR. This agrees with robust-Capon and worst-case analyses that tie the loading level to the uncertainty in the signal signature [45]–[47]. Flexible or

moored vertical arrays, whose geometry drifts, should therefore set the loading from a running SNR estimate.

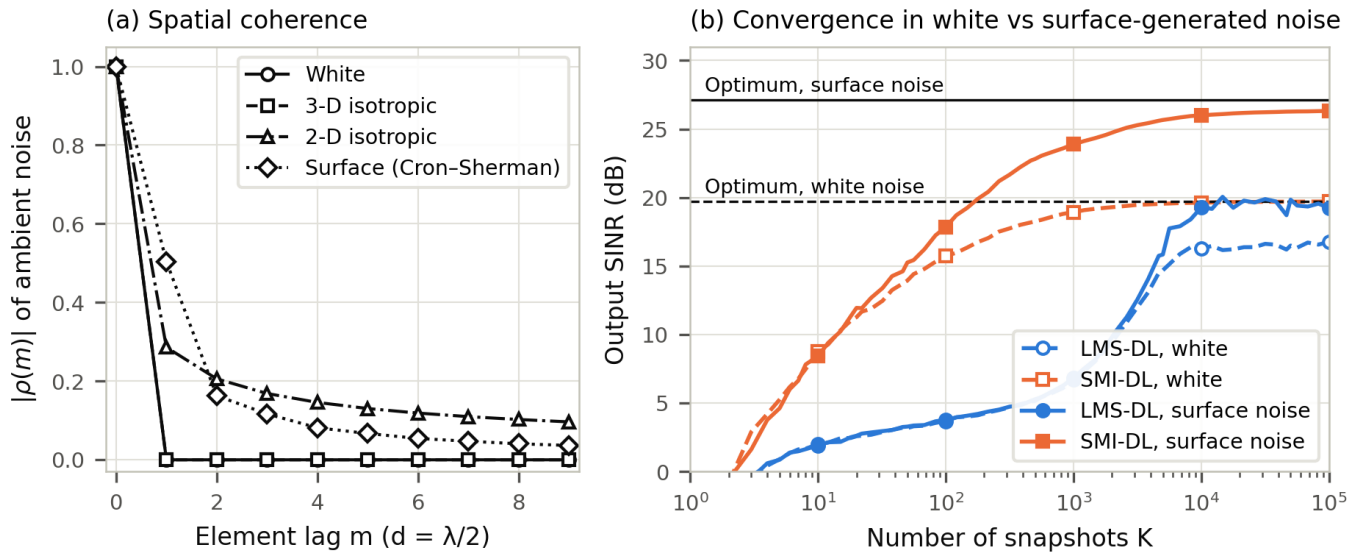


Figure 4. (a) Magnitude of the ambient-noise spatial coherence versus element lag. (b) Convergence of LMS-DL and SMI-DL in white (dashed, open markers) and surface-generated (solid, filled markers) noise, with the corresponding optima (100 trials).

D. Underwater Ambient-Noise Fields

Fig. 4 and Table IV show that the noise field changes both the achievable SINR and how much of it each algorithm can capture. At $d = \lambda/2$, 3-D isotropic noise has zero coherence at every nonzero lag, so it is indistinguishable from white noise. This holds only at the design frequency [13]: a wideband signal with a fractional bandwidth of 30–40% sees correlated noise away from the carrier. Cylindrical noise raises the optimum by 1.75 dB. Surface-generated noise raises it by 7.4 dB, because its

density vanishes at broadside, where the signal arrives [14]. SMI-DL recovers 6.4 dB of this gain by $K = 10^4$, but LMS-DL recovers only 2.5 dB, and the LMS-to-optimum gap widens from 3.0 dB to 7.9 dB. Two mechanisms explain this. The weak noise directions that must be resolved have the longest time constants in (18). In addition, by (17) the relative misadjustment loss grows with SINR_{opt} . Loading itself costs 0.76 dB of the surface-noise gain even at the Wiener point, because $\sigma_L^2 = 10\sigma_w^2$ masks the weak noise directions.

TABLE IV
SINR (dB) BY AMBIENT-NOISE FIELD (100 TRIALS)

Noise field	Optimum	Wiener point w_∞	LMS-DL, $K = 10^5$	SMI-DL, $K = 10^3$	SMI-DL, $K = 10^4$	Eigenvalue spread
White	19.74	19.72	16.77	18.93	19.62	1.0×10^5
3-D isotropic	19.74	19.72	16.31	19.02	19.64	1.0×10^5
2-D isotropic	21.49	21.44	17.48	20.41	21.32	1.5×10^5
Surface (Cron-Sherman)	27.11	26.35	19.23	23.89	26.00	4.0×10^5

Temporally colored noise has a negligible effect. Replacing the white sensor noise with the same $1/f$ process changes the SINR at $K = 10^4$ from 16.41 to 16.30 dB for LMS-DL and from

19.64 to 19.63 dB for SMI-DL. The loading level ($\sigma_L^2 = 10\sigma_w^2$) dominates the noise subspace, so temporal coloring of the noise, as opposed to the interference, is not a first-order effect for these receivers.

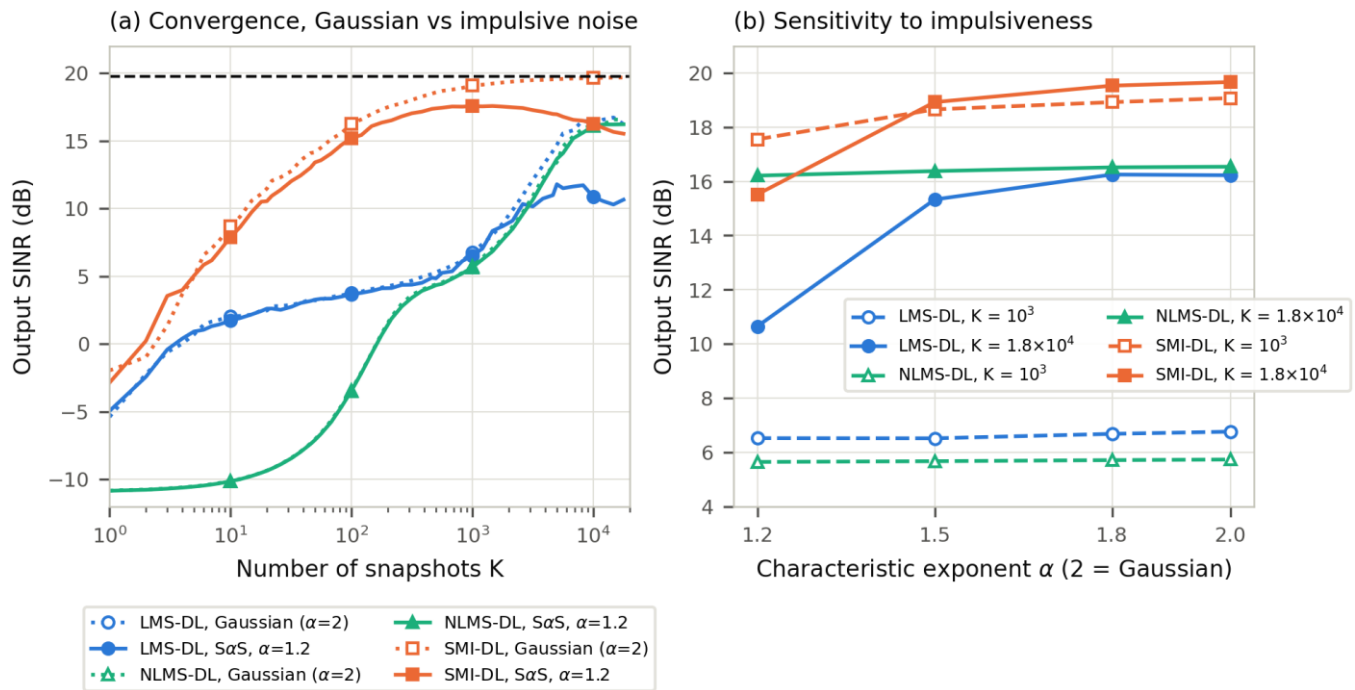


Figure 5. (a) Convergence in Gaussian noise (dotted, open markers) and S α S noise with $\alpha = 1.2$ (solid). (b) SINR versus characteristic exponent at $K = 10^3$ (dashed) and $K = 1.8 \times 10^4$ (solid). 100 trials; weights scored under the nominal Gaussian $\mathbf{S}_n$.

E. Impulsive Snapping-Shrimp Noise

Fig. 5 and Table V show that the power-averaged step of LMS-DL is fragile under impulsive noise. At $\alpha = 1.2$ it loses 5.6 dB, whereas NLMS-DL loses only 0.3 dB. The cause is the effective step $\mu_k \|\mathbf{x}(k)\|^2$. The recursive power estimate absorbs only a fraction $(1 - \beta)$ of an impulse, so the effective step can reach $\gamma/(1 - \beta) = 4$. This exceeds the single-step stability limit of 2, so one impulse can overshoot and kick the weights. The per-snapshot normalization (19) bounds the same

quantity by γ [38]. SMI-DL is the best performer in Gaussian noise but degrades as it averages longer. At $\alpha = 1.2$ it peaks at 17.6 dB near $K \approx 1.5 \times 10^3$ and falls to 15.5 dB by $K = 1.8 \times 10^4$, because the sample covariance of an infinite-variance process does not converge and a few large impulses act as a large random load. Robust scatter estimators such as spatial sign, Tyler's M-estimator, or fractional lower-order statistics [56], [58]–[61] are the natural remedies, and their combination with stochastic-gradient updates is a promising direction.

TABLE V
SINR (dB) UNDER SAS NOISE (100 TRIALS)

α	LMS-DL, $K = 5 \times 10^3$	LMS-DL, $K = 1.8 \times 10^4$	NLMS-DL, $K = 1.8 \times 10^4$	SMI-DL, $K = 10^3$	SMI-DL, $K = 1.8 \times 10^4$
2.0 (Gaussian)	14.69	16.22	16.54	19.07	19.67
1.8	14.56	16.25	16.52	18.92	19.53
1.5	14.45	15.33	16.38	18.66	18.93
1.2	11.79	10.65	16.21	17.55	15.52

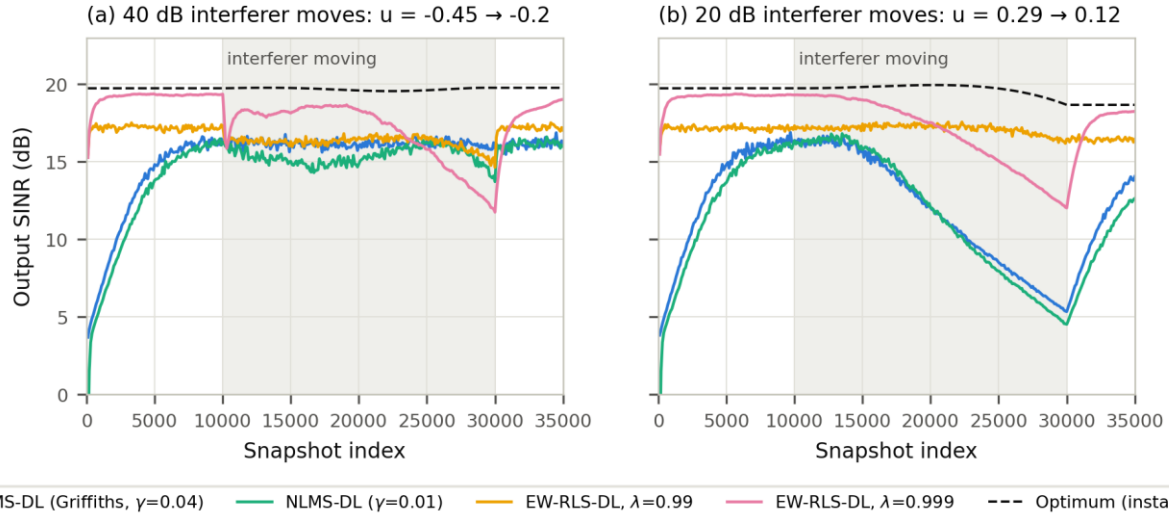


Figure 6. SINR versus snapshot index when one interferer moves linearly during snapshots 10^4 to 3×10^4 (shaded): (a) the 40 dB interferer, $u = -0.45 \rightarrow -0.2$; (b) the 20 dB interferer, $u = 0.29 \rightarrow 0.12$. The dashed line is the instantaneous optimum (100 trials).

F. Tracking a Moving Interferer

Fig. 6 shows that the tracking ability of LMS-DL depends on interferer power. When the 40 dB interferer moves, LMS-DL keeps a mean SINR of 16.2 dB with a worst-case loss of 4.1 dB relative to the instantaneous optimum, which is comparable to EW-RLS-DL with $\lambda = 0.99$ (16.3 dB, 5.1 dB). When the 20 dB interferer moves, the worst-case loss of LMS-DL grows to 13.3 dB (mean 11.8 dB), and NLMS-DL behaves similarly (14.2 dB). EW-RLS-DL with $\lambda = 0.99$ loses at most 3.0 dB (mean 17.1 dB). The asymmetry follows from (18). The 40 dB mode relaxes

in about 25 snapshots, but the 20 dB mode needs about 2.5×10^3 , during which the source moves 0.02 in u , comparable to the null width. RLS whitens the input, so its tracking speed is set by λ alone [40]. The usual trade-off appears: $\lambda = 0.999$ gives the best static SINR (19.3 dB) but the largest transient loss (8.0 and 6.6 dB), while $\lambda = 0.99$ tracks well but carries a 2.6 dB estimation loss. At 1000 snapshots/s, case (b) corresponds to about $0.5^\circ/\text{s}$, a plausible rate for a mobile node such as an AUV or glider operating nearby.

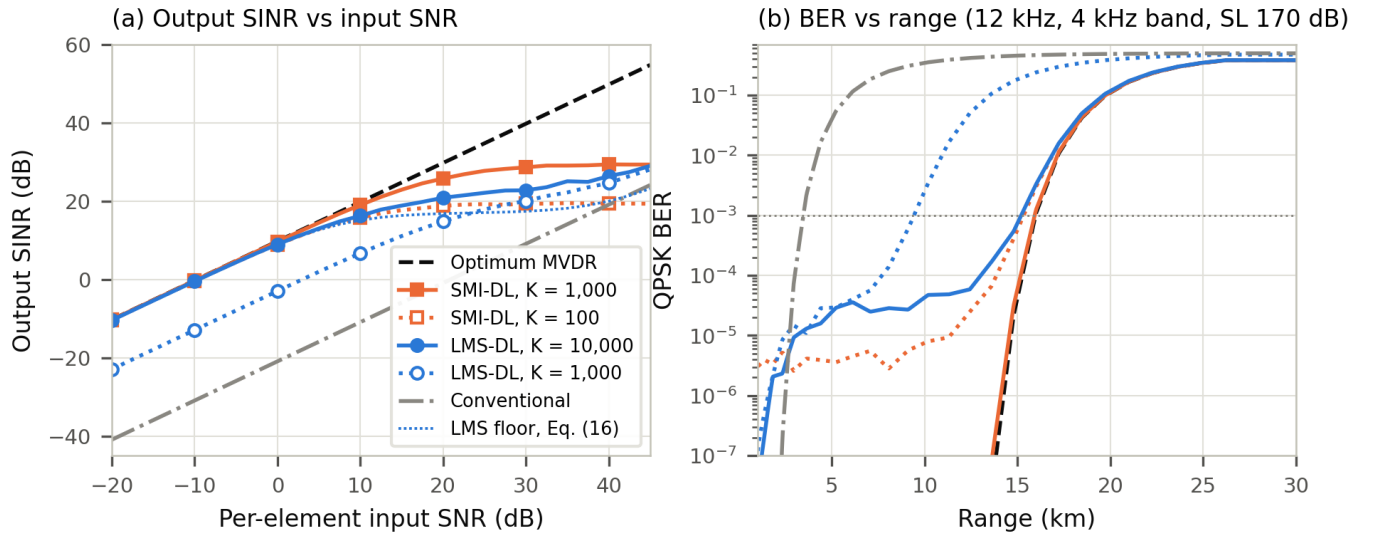


Figure 7. (a) Output SINR versus per-element input SNR (mean of 2000 trials), with the LMS floor (16). (b) QPSK BER versus range at $f_c = 12$ kHz, $B = 4$ kHz, $SL = 170$ dB re 1 μPa at 1 m, $k = 1.5$, with Thorp absorption and Wenz noise; interferer INRs held at 20/40 dB.

G. Link-Level Performance Versus Range

Fig. 7 and Table VI translate these effects into link performance. At $\text{BER} = 10^{-3}$, spatial nulling extends the range from 3.42 km with the conventional beamformer to 15.19 km with converged LMS-DL, against 15.96 km for the optimum. Near the range limit the SNR is low, every adaptive scheme is

noise-limited, and all cluster within 0.8 km of the optimum. At shorter range they separate. The optimum SINR rises 1 dB per dB of SNR, but the median SINR of LMS-DL saturates at 18–19 dB, consistent with the $2/\gamma$ ceiling of (17). The resulting BER floor of $2\text{--}5 \times 10^{-5}$ at 5–11 km is set by the lower tail of the SINR distribution. SMI-DL also saturates, because the signal is present in its training data [41]. With $K = 10^3$, however, its

ceiling of about 27 dB is high enough to remove the floor. For a network, an LMS-DL node close to its peer therefore cannot

reach the error rates that higher-order modulation requires, even when the link budget would allow it.

TABLE VI
LINK-LEVEL RESULTS (2000 TRIALS)

Receiver	Range at BER = 10^{-3}	Median SINR, SNR = 20 dB	Median SINR, SNR = 30 dB	BER floor, 5–11 km
Optimum MVDR	15.96 km	29.7 dB	39.7 dB	none
SMI-DL, K = 10^3	15.91 km	25.4 dB	27.3 dB	$< 10^{-7}$
SMI-DL, K = 10^2	15.28 km	17.3 dB	17.7 dB	$\approx 4\text{--}9 \times 10^{-6}$
LMS-DL, K = 10^4	15.19 km	18.2 dB	18.5 dB	$\approx 2\text{--}5 \times 10^{-5}$
LMS-DL, K = 10^3	9.38 km	14.8 dB	18.2 dB	BER $< 10^{-3}$ only below 9.4 km
Conventional	3.42 km	−0.9 dB	9.1 dB	interference-limited

VI. IMPLICATIONS AND LIMITATIONS

The results translate into design guidance for UWA array receivers. First, the step size should be chosen from the target SINR through (17) rather than fixed, and a variable step-size rule [39] can combine fast initial convergence with a low floor. Second, the loading should track the SNR when the array geometry is uncertain. Third, per-snapshot normalization is a one-line change that makes the stochastic-gradient beamformer far more robust to snapping-shrimp noise. Fourth, RLS or a hybrid scheme is preferable when weak co-channel users move.

Convergence time also matters at the network level. At symbol rates of 1–4 kbaud, the 10^4 snapshots that LMS-DL needs correspond to 2.5–10 s. Measured UWA channels can change on time scales of seconds or less, especially with platform motion [7], [8]. Block methods such as SMI-DL, which converge in a few hundred snapshots, are therefore better matched to mobile networks, and their $O(N^3)$ cost is modest for arrays of about ten elements.

The study has limitations that define future work. The model is narrowband and single-path. Wideband processing [21], [55] and multipath arrivals coherent with the signal, which cause signal cancellation in MPDR-type beamformers [42], must be added. Doppler and platform motion [20] are not modeled, and all results are simulated, so validation with recorded at-sea array data, as in [22], [23], is needed. Finally, comparisons with robust covariance estimators [61] and with data-driven beamformers [24] would complete the picture.

VII. CONCLUSION

This paper analyzed the diagonally loaded Griffiths LMS beamformer for underwater acoustic arrays. A closed-form approximation shows that its steady-state SINR is limited by output-power-driven misadjustment and saturates at $10\log_{10}(2/\gamma)$ dB, and Monte Carlo simulation confirms it to within 0.75 dB. LMS-DL suppresses strong interferers quickly but converges slowly on weak ones. It captures only 2.5 dB of the 7.4 dB gain offered by surface-generated noise, loses 5.6 dB in impulsive noise unless the step is normalized per snapshot, and cannot track weak moving interferers. At link level, it attains 95% of the optimum range at BER = 10^{-3} but leaves an error floor at short range. SNR-dependent step size and loading,

per-snapshot normalization, and block or RLS processing for mobile scenarios are the main design recommendations. Wideband and multipath extensions with at-sea validation are the next steps.

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CONFLICT OF INTEREST

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DECLARATION OF GENERATIVE AI USE

An AI assistant (Claude, Anthropic) was used to support simulation coding, language editing, and literature organization. The author developed the final code, generated all figures from their own data, and reviewed and verified all content. The author takes full responsibility for the work data.

AVAILABILITY

The MATLAB and Python code that reproduces all figures and tables in this paper is available from the corresponding author upon reasonable request.

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Sehrish received the M.S. degree in telecommunication networks from the University of Engineering and Technology (UET), Lahore, Pakistan, and the B.S. degree in telecommunication engineering from Government College University (GCU) Faisalabad, Pakistan. She is a practicing engineer. Her research interests include cross-layer protocol optimization for underwater acoustic sensor networks, with attention to energy efficiency and reliable data delivery in sparse and mobile deployments. She works with MATLAB, Python, and C++.

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