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Formation Control of Unmanned Vehicles via Extended Feeedback Consensus

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Abstract—This paper has considered the problem of formation control of multiple quadrotor using a diversified feedback linearization technique. Taking the advantage of double integrator which normally can achieve any unrestrained shape, we have devised a linearization technique that is made possible using extended feedback. The technique has the ability of transmuting the dynamics of a quadrotor's reference point to four double integrators in accordance with yaw angle and position of quadrotor in the space. The traditional exact feedback technique require a jerk (which is the derivative of acceleration) but it is not an issue for the extended feedback method. Ending by the conclusion the section numerical example elucidate formation control of quadrotor by using reference point and center of masses of the quadrotors.

Keywords—Cooperative control, Consensus, Formation, Extended Feedback, Multi-agents

I. Introduction

Due to large number of researcher attraction to the field of Cooperative or distributed control of multiagent system, the importance of Formation control reached to an undeniable level [1] [2]. And this became possible due to consensus in the formation of multiagent system, which mean that the agents has to coincide on a state or issue. For the formation of arbitrary shaped structure Consensus algorithms are very applicable. Like in homogenous linear systems the agent get-into a formation via the relative distances among multiple agents [3]. However for heterogeneous systems, it could be achieved by using the position of each and every (individual) agent [4].

The linearization over exact feedback [5] technique can be employed to all agents where they have non-linear dynamics like mobile robots on ground and by using this technique the dynamics are linearized to into integrators. In fact the exact feedback linearization could possibly be executed over a quadrotor [6] which leads to such dynamics whose integrators are quadruple. Some time it is very difficult to handle a system, possessing large number of integrators as employability of integrators leads to decrease in gain margin and aggravate the lag (in phase). Additionally jerk is a necessary element of quadruple integrators whereas the derivative of acceleration is jerk. In general there is no direct method for determining the jerk also it couldn't be computed properly from the estimated value of sensor. Because of these issues we have devised a linearized extended feedback technique which is employed for controlling the formation a network of positive systems. In this technique we have to consider a predetermined point of reference for all quadrotors across which the linearization of model is done. And as a result double integrated model is achieved where no jerk is required. Possibly this linearization technique could be applied to any other formation control framework.

An attempt for controlling the formation of quadrotors via consensus has been done here in this paper. The formation is achieved by using our extended feedback linearization technique. Double integrators are achieved from the natural dynamics of quadrotors by using our diversified technique [7] i.e. extended feedback linearization. As it can be seen in the numerical examples, that the quadrotors' center of mass and reference points achieved a proper shape called formation in the given space.

Notation: I_n represent $\boldsymbol{n} \times \boldsymbol{n}$ unit matrices while $\otimes$ signify the Kronecker product's operator.

II. The Issue of Consensus

The issue of consensus [3] is considered in this section, as we are going to address formation control of double integrators as agents.

A. Double Integrator Model

Let assume an agent or vehicle is given by:

$$\begin{bmatrix} \dot{z}_{i,j} \\ \ddot{z}_{i,j} \end{bmatrix} = A_{veh} \begin{bmatrix} z_{i,j} \\ \dot{z}_{i,j} \end{bmatrix} + B_{veh} v_{i,j}, \quad (1)$$

Where, $[\![z_{i,j} \quad \dot{z_{i,j}}]\!]^T \in \mathbb{R}^2$ is the state, $v_{i,j} \in \mathbb{R}$, is the input

$$A_{veh} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \quad B_{veh} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

for ∈ {1,. . . . , N} . The dynamics of all agents are same and the entire system of N agents could be given for every single agent as given in Eq. (2).

$$\dot{z}_j = A_j z_j + B_j v_j \quad (2)$$

Where $z_j = [z_{1,j},\ \dot{z_{1,j}},\ \ldots z_{Nj},\ \dot{z}_{n,j}]^T$ is the state

B. Designing the Controller

We have taken into account, the controller for the whole system given by (2) has the following controller

$$v_j = F_j L(z_j - r_j) \quad (3)$$

While $r_j \in \mathbb{R}^{2N}$ is the desired shape of formation which could be given by $r_j = [r_{1,j}, 0, \ldots. r_{N,j}, 0]^T$. It is obvious from the r_j matrix that the shape will not be changed as there are Zeroes in the r_j matrix. $L = L_G \otimes I_2$ *where* $L_G \in \mathbb{R}^N$ represents a network topology of multiple robots that is generally called Graph Laplacian. $F_j = I_N \otimes F_{veh}$ is perpetual gain where $F_{veh} = [f_{1j}\ \ f_{2j}]$. As a result we acquire a closed loop system using (2) and (3) such as

$$\dot{z} = A_j z_j + B_j F_j L \otimes I_2(z_j - r_j). \quad (4)$$

The system defined by (2) and L_G provides base for the *Lemma 2.1* given below.

Lemma 2.1: [3] The formation $r_j, z_j \longrightarrow\rightarrow r_j$ is achieved by the entire system (2) iff L_G has a zero eigenvalue having algebraic multiplicity 1 and

$$A_{veh} + \lambda B_{veh} F_{veh} \quad (5)$$

is Hurwitz where each nonzero eigenvalue of L (that are referred by λ) and has the value as $\alpha_j + \beta_j\sqrt{-1}$.

As the Eq. (5) is Hurwitz, hence the conditions (for achieving the formation) of F_i could be stated as (below):

$$\begin{aligned} f_{1,j} &< 0, \quad f_{2,j} < 0 \\ \frac{f_{2,j}^2}{f_{1,j}} &< -\frac{\beta_j^2}{\alpha_j(\alpha_j^2 + \beta_j^2)}. \end{aligned}$$

III. LINEARIZING THE FEEDBACK OF QUADROTOR

Quadrotor dynamics could be given by the following set of equation as in (6)

$$\begin{aligned} \begin{bmatrix} \ddot{x}_i^o \\ \ddot{y}_i^o \\ \ddot{z}_i^o \end{bmatrix} &= \frac{1}{m} R(\psi_i, \theta_i, \phi_i) \begin{bmatrix} 0 \\ 0 \\ U_{i,1} \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ g \end{bmatrix}, \\ \ddot{\phi}_i &= \dot{\theta}_i \dot{\psi}_i \frac{I_y - I_z}{I_x} - \frac{I_r}{I_x} \dot{\theta}_i \Omega_i + \frac{d}{I_x} U_{i,2}, \\ \ddot{\theta}_i &= \dot{\phi}_i \dot{\psi}_i \frac{I_z - I_x}{I_y} - \frac{I_r}{I_y} \dot{\phi}_i \Omega_i + \frac{d}{I_y} U_{i,3}, \\ \ddot{\psi}_i &= \dot{\theta}_i \dot{\phi}_i \frac{I_x - I_y}{I_z} + \frac{1}{I_z} U_{i,4} \end{aligned} \quad (6)$$

Where, $(x_o^{\ i}, y_o^{\ i}, z_o^{\ i}) \in X \times Y \times Z$ is mass center position, while Euler angle is given by $(\psi_i, \theta_i, \varphi_i)$, i.e. (yaw, pitch,roll). The mass is represented by m, distance between motor and mass center is referred by d, inertia of rotor is I_r, while the inertia for each axis of whole quadrotor is (I_x, I_y, I_z), and finally the rotation of matrix is reffered by $R(\psi_i, \theta_i, \varphi_i)$. The control input for a quadrotor is $U_i = [U_{i,1}, U_{i,2}, U_{i,3}, U_{i,4}]^T \in R^4$ *where* $U_{i,1}$ is the thrust of rotor and $U_{i,2}$, $U_{i,3}$ and $U_{i,4}$ are the torques in the respective direction given by X, Y, and Z *axis.* However $\omega_{i,1}$, $\omega_{i,2}$, $\omega_{i,3}$ *and* $\omega_{i,4}$ corresponds to the angular velocity of corresponding rotors as is obvious from the following equation.

$$\Omega = -\omega_{i,1} + \omega_{i,2} - \omega_{i,3} + \omega_{i,4}$$

A. Extended Feedback Linearization

This subsection presents an extended feedback linearization technique [7] for quadrotors. We have introduced a reference point $(x_i, y_i, z_i) \in R \times R \times R$ which is at a fixed distance h from the center of mass of quadrotor as given by Fig. 1. By choosing the reference above center of mass will lead to the following equation. i.e. Eq. (7).

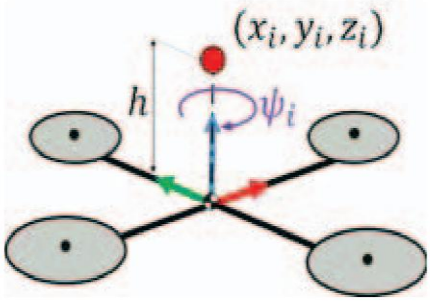

Fig. 1. The reference point above quadrotor

$$\begin{bmatrix} x_i \\ y_i \\ z_i \end{bmatrix} = \begin{bmatrix} x_i^o \\ y_i^o \\ z_i^o \end{bmatrix} + R(\psi_i, \theta_i, \phi_i) \begin{bmatrix} 0 \\ 0 \\ h \end{bmatrix}. \quad (7)$$

By taking the second order derivative of Eq. (7) with respect to time and after substituting the result into Eq. (6) we get Eq. (8) as below.

$$\begin{bmatrix} \ddot{x}_i \\ \ddot{y}_i \\ \ddot{z}_i \\ \ddot{\psi}_i \end{bmatrix} = F(\psi_i, \dot{\psi}_i, \theta_i, \dot{\theta}_i, \phi_i, \dot{\phi}_i) + G(\psi_i, \theta_i, \phi_i) U_i \quad (8)$$

where $F : R \times R \times R \times R \times R \times R \rightarrow R^{4\times 1}$ and $G : R \times R \times R \rightarrow R^{4\times 4}$ are nonlinear complicated matrices. Afterwards a linearization feedback law has been presented such as

$$\begin{aligned} U_i &= G^{-1}(\psi_i, \theta_i, \phi_i)\left(-F(\psi_i, \dot{\psi}_i, \theta_i, \dot{\theta}_i, \phi_i, \dot{\phi}) + v_i\right) + q_i \\ &\forall(\psi_i, \dot{\psi}_i, \theta_i, \dot{\theta}_i, \phi_i, \dot{\phi}) \in \mathbb{R} \times \mathbb{R} \times \mathbb{R} \times \mathbb{R} \times \Phi \times \mathbb{R} \quad (9) \\ \Phi &:= \left\{\phi \in \mathbb{R} \setminus \{\phi_0\} : \phi_0 = \pm\frac{\pi}{2} + 2\pi k, (k = 1, 2, \ldots)\right\} \end{aligned}$$

where $v_i = [v_{ix}, v_{iy}, v_{iz}, v_{i\psi}]^T \in \mathbb{R}^4$ and $q_i = [q_{i1}, q_{i2}, q_{i3}, q_{i4}]^T \in \mathbb{R}^4$

$$q_{i,1} = q_{i,4} = 0, q_{i,2} = k_{\phi_i}\left(\frac{I_x}{d}\right)\dot{\phi}_i, q_{i,3} = k_{\theta_i}\left(\frac{I_y}{d}\right)\dot{\theta}_i$$

Note that $k_{\varphi i}$ and $k_{\theta,i}$ are unvaried gains with negative values. The dynamics of quadrotor's reference point (8) are transformed to double integrators by employing feedback law (9), which results in a linearized dynamics for every individual reference point as given by Eq. (10)

$$\ddot{\mathsf{z}}_{i,j} = v_{i,j},\ j \in \{x, y, z, \psi\}. \tag{10}$$

B. Linearization of Exact Feedback

This subsection presents the linearization method [5] of exact feedback scenario which is traditionally being used for quadrotors. As the work [5] generated fully independent result from the results given by [6], then by using Eq. (6), we have an expression of the dynamics of the quadrotor with respect to $(x_i^0, y_i^0, z_i^0, \psi_i^0)$ that could be given as

$$\begin{bmatrix} \ddot{x}_i^o \\ \ddot{y}_i^o \\ \ddot{z}_i^o \\ \ddot{\psi}_i \end{bmatrix} = A_{i,q,1}(\psi_i, \dot{\psi}_i, \theta_i, \dot{\theta}_i, \phi_i) \begin{bmatrix} U_{i,1} \\ U_{i,2} \\ U_{i,3} \\ U_{i,4} \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ \frac{g}{m} \\ 0 \end{bmatrix} \tag{11}$$

where $A_{i,q,1} \in \mathbb{R} \times \mathbb{R} \times \mathbb{R} \times \mathbb{R} \times \mathbb{R} \rightarrow \mathbb{R}^{4\times 4}$ is a nonlinear matrix. Taking second order derivative of Eq. (11) while excluding the dynamics of ψ_i term with respect to time and again considering Eq. (6) we get Eq. (12) as

$$\begin{bmatrix} \ddddot{x}_i^o \\ \ddddot{y}_i^o \\ \ddddot{z}_i^o \\ \ddot{\psi}_i \end{bmatrix} = A_{i,q,2}(U_{i,1}, \dot{U}_{i,1}, \psi_i, \dot{\psi}_i, \theta_i, \dot{\theta}_i, \phi_i, \dot{\phi}_i) \begin{bmatrix} \ddot{U}_{i,1} \\ U_{i,2} \\ U_{i,3} \\ U_{i,4} \end{bmatrix} \tag{12}$$
$$+ F_{i,q}(U_{i,1}, \dot{U}_{i,1}, \psi_i, \dot{\psi}_i, \theta_i, \dot{\theta}_i, \phi_i, \dot{\phi}_i)$$

Where $A_{i,q,2} \in \mathbb{R} \times \mathbb{R} \times \mathbb{R} \times \mathbb{R} \times \mathbb{R} \times \mathbb{R} \times \mathbb{R} \times \mathbb{R} \rightarrow \mathbb{R}^{4\times 4}$ and $F_{i,q} \in \mathbb{R} \times \mathbb{R} \times \mathbb{R} \times \mathbb{R} \times \mathbb{R} \times \mathbb{R} \times \mathbb{R} \times \mathbb{R} \rightarrow \mathbb{R}^{4\times 1}$ are non-linear complex matrices. Then by commencing a feedback linearization law we have

$$\begin{bmatrix} \ddot{U}_{i,1} \\ U_{i,2} \\ U_{i,3} \\ U_{i,4} \end{bmatrix} = A_{i,q,2}^{-1}\left(u_i - F_{i,q}\right) \tag{13}$$

Where $u_i = \left[u_{i,x}, u_{i,y}, u_{i,z}, u_{i,\psi}\right]^T \in \mathbb{R}^4$ is actually the input for (x^0, y^0, z^0, ψ) while $A_{i,q,2}$ is nonsingular except $U_{i,1} = 0$. And in this manner we acquire dynamics of quadrotor in a linearized form as given by

$$\ddddot{\mathsf{z}}_{i,j} = v_{i,j},\ j \in \{x, y, z, \psi\}.$$

IV. Numerical Example

The quadrotor has several physical parameters that are enlisted in TABLE I. For the sake of convenience we considered 8 quadrotor i.e. *N=8*, whereas the distance h is taken as *0.1 [m]*. For the same calculation the values of gain are $f_{1,j} = -0.1$, $f_{2,j} = -1$ and $k_\theta = k_\varphi = -10$. And the graph Laplacian is

$$L_G = \begin{bmatrix} 2 & -1 & 0 & 0 & 0 & 0 & 0 & -1 \\ -1 & 2 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 2 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 2 & -1 \\ -1 & 0 & 0 & 0 & 0 & 0 & -1 & 2 \end{bmatrix}$$

and the topology of multiple quadrotor network is shown in Fig. 2 in the form of undirected graph. Quadrotor is denoted by every circle bearing a number while edge signify the communication between the quadrotors. Lemma 2.1 satisfies the given gains and L_G. However the desired shape is a $2 \times 2 \times 2$ cube such as

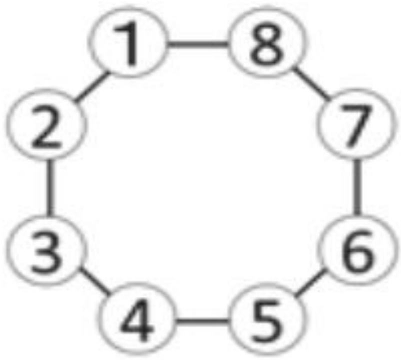

Fig. 2. Quadrotors' Communication Network

$$r_x = [3,0,3,0,1,0,1,0,3,0,3,0,1,0,1,0]^\top,$$
$$r_y = [1,0,3,0,3,0,1,0,1,0,3,0,3,0,1,0]^\top,$$
$$r_z = [1,0,1,0,1,0,1,0,3,0,3,0,3,0,3,0]^\top,$$
$$r_p = [0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0]^\top.$$

The incipient position are $x(0) = [0.5,3,1,2,3,3.5,2,1]^T$, $y(0) = [3,2,1,3,2,3.8,1,0.2]^T$, $z(0) = [3,2,1,3,1,2,3.5,0.5]^T$ and $\psi(0) = \left[0, \frac{\pi}{2}, 0, \pi, 0, \pi, 0, \frac{\pi}{2}\right]^T$. The starting values of Euler's angles can be given as $\theta(0) = \phi(0) = [0,0,0,0,0,0,0,0]^T$, while the initial linear velocities and initial angular velocities are taken zero.

Using the above mentioned parameters and their values we simulated formation control starting from t=0s till t=120s using the extended feedback linearization as obvious from Figs. 3-5 and Fig. 6 shows the time responses of mass centers plus yaw angles. After applying the extended feedback linearization techniques it can be shown that both reference point as well as yaw angles have achieved the formation. In the same way 3D shots of center of masses are shown in Fig. 7-9 and the respective time response are revealed in Fig. 10 from which we can deduce that the formation is achieved by yaw angles and center of masses as well.

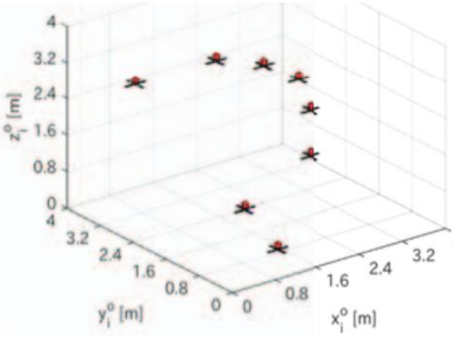

Fig. 3. 3D shot of quadrotors at $t = 0$ [s] (Extended linearization)

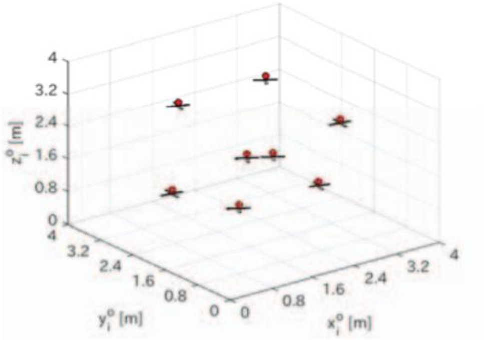

Fig. 4. 3D shot of quadrotors at $t = 20$ [s] (Extended linearization)

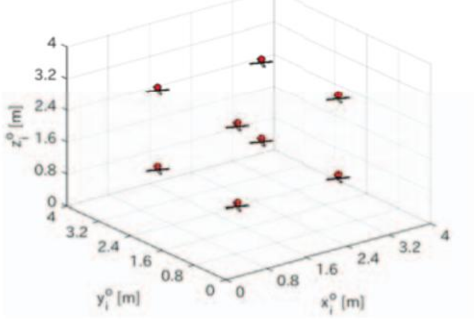

Fig. 5. 3D shot of quadrotors at $t = 120$ [s] (Extended linearization)

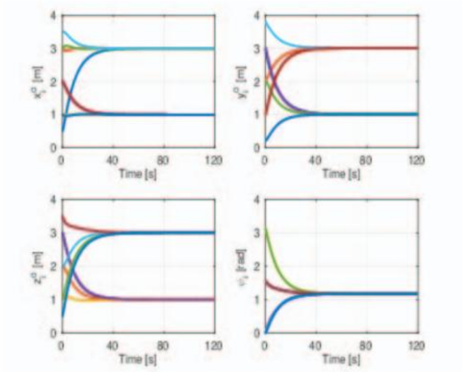

Fig. 6. Time response of positions and yaw angles of each quadrotor (Extended linearization)

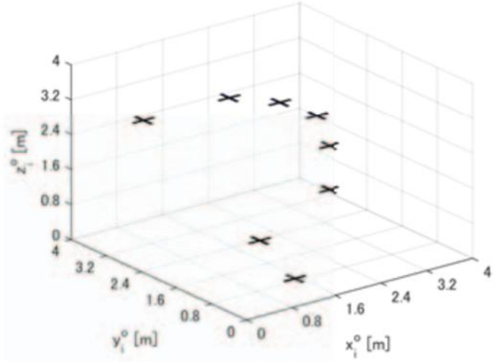

Fig. 7. 3D shot of quadrotors at $t = 0$ [s] (Exact linearization)

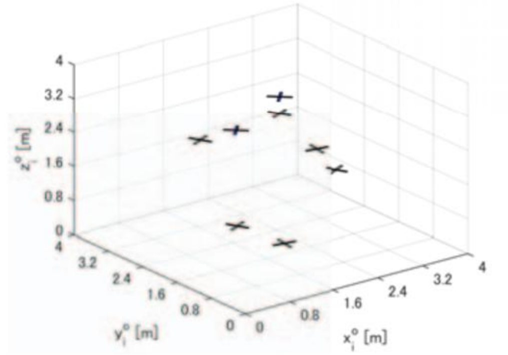

Fig. 8. 3D shot of quadrotors at $t = 20$ [s] (Exact linearization)

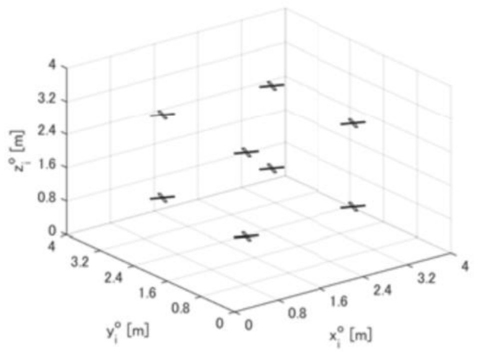

Fig. 9. 3D shot of quadrotors at $t = 120$ [s] (Exact linearization)

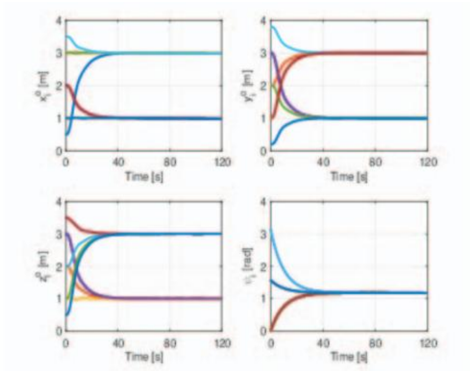

Fig. 10. Time response of positions and yaw angles of each quadrotor (Exact linearization)

CONCLUSION

To overcome the mentioned problem we developed an additional technique that is obtained from the linearization of nominal dynamics of a quadrotor, which is done to achieve double integrators that are beneficial for achieving the formation of multiple vehicles via consensus. This feedback technique is appropriate for the double integrators (bearing consensus issue), where the UAVs can get their desired shape or formation without going into the space. Unlike the mobile ground robots where such characteristics are not explicitly available in their literature, and where the vehicles can't achieve formation without movement. However the conventional linearization method of exact feedback resolve this issue for quadruple integrator, but the linearized extended feedback method is more attractive in many cases than the exact linearization such as response of the system to frequency.

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