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Investigating Future Consumptionf Electricity Estimates in Pakistan using Linear Regression Analysis

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Abstract**—** In this study, an inclusive effect of economy, gross domestic product (GDP) and other socio-economic terms are deeply taken into account for long lasting and future projection of electricity usage in Pakistan. The explaining variables considered in this study are gross domestic product GDP, per capita GDP, electricity consumption price/kwh and population, to develop the different regression models. The historical data considered for time period of past 47 years from 1970-2016. This paper is divided into two parts, in first part estimation of income, price and GDP elasticities are evaluated for residential, non-residential and total electricity consumption models. These elasticities showed that long-run and short-run price elasticities for domestic model are -0.30 and -0.32, for non-domestic model 0.42 and 0.64 where-as for total electricity consumption model price elasticities are 0.50 and o.93. In addition GDP/per capita and GDP resulted higher values. In second part, different statistical models are presented by using linear regression, which are based on the stationary or co-integrated time series data. Moreover for checking the validity of proposed models different statistical tests are conducted. A comparison with available national forecasts, which are proposed through different econometric models, like support-vector model or Pakistan's long range-energy alternative plaining (LEAP) model was exmined, resulting that proposed regression model has compatibility with national projections, with deviation of ±2% to ±12% for the best and worst case, these deviations are acceptable in the time span taken into account.

Keywords**—** Forecasting, Electricity consumption Estimates, Multiple linear regression.

I. Introduction

Energy has major contribution in the development and better economic growth of any country. Growing demand for energy needs to be accurately estimated, to maintain the socioeconomic development and hence required to construct quick-witted forecasting models and algorithm. Prediction of energy consumption pattern based on non-economic and economic index may be accomplished by non-linear or linear statistical methods or by simulation models. Worldwide energy consumption is increasing fast (2.2% reported in the year 2017)[1] due to continuously changing living standards, increase in human population, focus on large scale industrialization and increasing rate of urbanization in the developing countries[2]. Pakistan is also a developing country and has registered a growth rate of 6.1% in its energy consumption in the year 2017 [1].There are six main sectors of electricity consumption, as illustrated in fig 1 percentage-wise sectorial electricity consumption of Pakistan([3][4]

Modeling and forecasting of electricity consumption is an important planning activity which needs to be conducted on regular basis to incorporate the dynamics of socioeconomic variables and policies that directly impact the electricity consumption profile of the country.

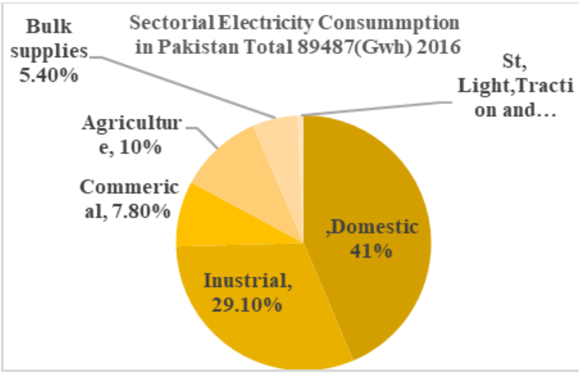

Fig1 Percentage-wise sectorial Electricity consumption of Pakistan (NTDC)[4]

Currently energy modeling is an interesting subject among the engineers and scientists concerned with the problem of energy supply and demand. Energy modeling in every sector assemble significant contribution in policy formation and planning, regarding to this energy management and planning can't possible without the information of past, present trend and future projection of energy demand. Future forecasting of energy demand helps to long-term policy makers in developing countries [5]. Lower estimation of consumption would lead to create shortfall between supply and demand, which may cause of deviation in economic growth rate, whereas higher estimation may cause of wasted financial resources due to unnecessary investment. Therefore it is desirable to propose energy consumption model with better accuracy for the best use of resources. Also it would be desirable that proposed model must handle the non-linear data of indicating variables [6]. It is important to understand the impact of co-related variables which are effecting electricity consumption. Some authors have studded the behavior of electricity consumption drivers which are essential for forecasting. Review of published studies by national and international authors are discussed here. Tabasi, et al, [7] studied the behavior of indicating parameters and performed forecasting up-to 2022, by evaluating historical data from (2003-13), for Germany in the year of (2016) using linear regression analysis. Salari and Javid, (2016)[8] investigated the indicator of energy consumption and presents the alternative scenarios for reducing the residential energy demand of United States by using both static and dynamic estimation models. Mohamed and Bodger,[9] performed an critical analysis over correlation between the electricity usage and its demographical variables, and estimated the future energy consumption by using multiple linear regression analysis for New Zealand. S. Saravanan et al, [10] studied the influence of economical deviations on energy usage and investigated the future estimates of electricity consumption up-to (2021) in India with the help of Artificial neural network (ANN) and linear regression. M. Kankal and M. Ihsan, [5] considered gross domestic product (GDP), population import and export as explaining variables for electricity usage to analyze the correlation and investigate the future consumption in Turkey, they developed linear regression and (ANN) models. Bianco et al., (2009)[2] proposed linear regression model to analyze the impact of growing population , GDP, and electricity price per kwh over electricity consumption in Italy, and they performed forecasting up-to 2030 on the basis of 37 years historical data. The study caried out by same authers of refference [10], (2010) [11] they developed the linear regression and Grey prediction models based on the data from (1975-2007) for Romania, where they examined the income and price elasticities, also forecasted the future growth of electricity consumption. Erkan et al, (2009) [12] measured short-run and long-run price and income elasticities and estimated the future demand of energy, through co-integration tests and Auto Regressive Integrated Moving Average (ARIMA) model. They used time series data from (1984-2007) and considered different independent variables in the case of Turkey.

Literature regarding electricity consumption and its elasticities estimations in Pakistan will be discussed in well manner. N. Hussain et al, (2018)[13] developed Pakistan's Long-range Energy Alternative Paining (LEAP) model, and considered data from (1991-2015) they analyzed the electricity usage and predicted future consumption up-to (2050). In some studies of (2014) and (2017) [14]15] ARIMA and Holt Winter methods are focused for examining the relationship between dependent and independent variables and for future projections of electricity consumption. Mostly LEAP model was used in different recent studies [16] [17] [18] for energy consumption forecasting in Pakistan. Mamodia (2015) [19] developed multiple linear regression technique to determine the impact of economic changes on electricity usage for India. Rallapalli and Ghosh (2017)[20] used ARIMA and Time Trend method in India to make forecasts and investigate the influence of economic factors which are gross domestic product, population, price of electricity import and export considered as demographical variables. Literature review of national authors comprising on estimation of income and price elasticities of electricity consumption briefly analyzed. Akbar Ullah et al, (2019)[21] presented Auto Regressive Distributed lag (ARDL) and Econometric decomposition models to calculate the price and income elasticities. They considered modeling data for the period of (1972-2012) of explanatory variables which are electricity price and income. The concluded survey of estimations [22][23][24] presents the Econometric and log linear function over correlated variables such as income, price and no of customers. (Faisal Mehmood et al,) (2019)[25] developed ARDL and modified IPAT models to measure the casual relationship between electricity consumption and indicating parameters (i-e price and income) They accounted the data for the period of (1976-2013).

Pakistan is blessed with many optimum resources like hydro, coal, thermal, solar and wind energy, but unfraternally unable to meet its own demand only because of few reasons such as Government's poor future policy formation, planning and management and political interference, because every political party only think about its tenure. According to the best knowledge of authors' literature concerned with Pakistan is facing the problem of scientific analysis of demand drivers and estimation of price and income elasticities for non-domestic and domestic consumption. Different national authors mostly used different models other than statistical method named as multiple linear regression analysis which has an application of quick-weighted with better accuracy and easy to implement. The beauty of this study is to investigate the behavior of co-related variables and to provide the future prediction of electricity consumption in Pakistan for long-term policy formation using different econometric models, based on historical stationary data and its cointegration tests. In the first part of paper it is targeted to estimate price and income elasticities of electricity consumption. Where-as in second part the target is to propose forecasting models, with better accuracy. By using multiple linear regression, GDP, population and electricity price/kwh are selected as explaining variables to forecasts Pakistan's electricity consumption up-to 2040 is presented. For simplification of proposed regression models only income [ratio between GDP and population (GDP per capita)] is also selected as explaining variable.

The results of proposed models compared with the Pakistan's official National Transmission and Dispatch company(NTDC) forecasts, and with public private research organization's results which are concerned with the field of energy management. It is illustrated that there is best understanding among the different predictions .

II. METHODOLOGY

A. *Data Sets*

Time series data for forecasting is considered for the period of 1970-2016, on the annual electricity consumption of past 47 years, according to usage (domestic and non domestic), data is collected from National Transmission and Dispatch Company Limited (NTDCL) report[4], NTDCL is a public-private company provides services throughout the coutry except Karachi region, acts as Transmission Network system operator; also deal with cross-border electricity trade.

Time series data of electricity consumption are reported in fig.2 and the demographic variables (i.e.population, GDP, GDP per capita and price of electricity per kwh) are presented in fig.3.

It can be noted that growth trend of consumption remained substantial linear. but from 2000-2010 a marked declines in electricity usage was noticed due to energy shortage of that period [26]

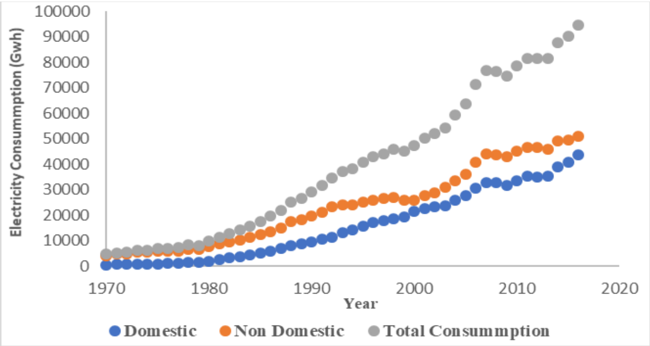

Fig.2.Represent historical data for electricity consumption

The historical data of GDP and GDP per capita is taken from the NTDCL report [4] if one compares the profile of Electricity consumption with GDP and population trend a percantage reduces in GDP and in response to that there is decrease in both domestic and non domestic consumption, so it can be observed that there is strong relation between GDP in fig.3.a, electricity consumption in fig.2. and population trend in fig.3.b. Around 2008 Electricity consumption grows faster because of three root causes, rapidly increasing GDP per capita, faster population growth rate and electricity intensity, which is the cause of summer air conditioning load, of mainly domestic and commercial costomers.

The time series data of population for same period is reported in fig.3.b.and it has linear trend through out the considered period with average growth rate of 2.965% and average death rate is 0.698% , the population data is collected from Pakistan Bureau of Statistics (PBS)[27], is the Government's offical institute which is concerned with country statistical record of population, labour force, empolyment,products, import and export etc. The data of electricity price reported in fig.3.d.it is distributed in three categories for the sake of understanding; domestic, non-domestic and total electricity price as shown in fig; data is collected from the NTDC[4] has interesting behavior slightly increasing up-to 1993 then increasing rapidly from1994-2016 because of different economic cycles, .and higher increments was detected in non-domestic tariff, in 2016 it reaches at peak value, the variations in oil and gas prices were the main causes of these sudden changes. Pakistan is a developing country in which mostly generation of electricity is dependent on imported oil and gas resources.[28]

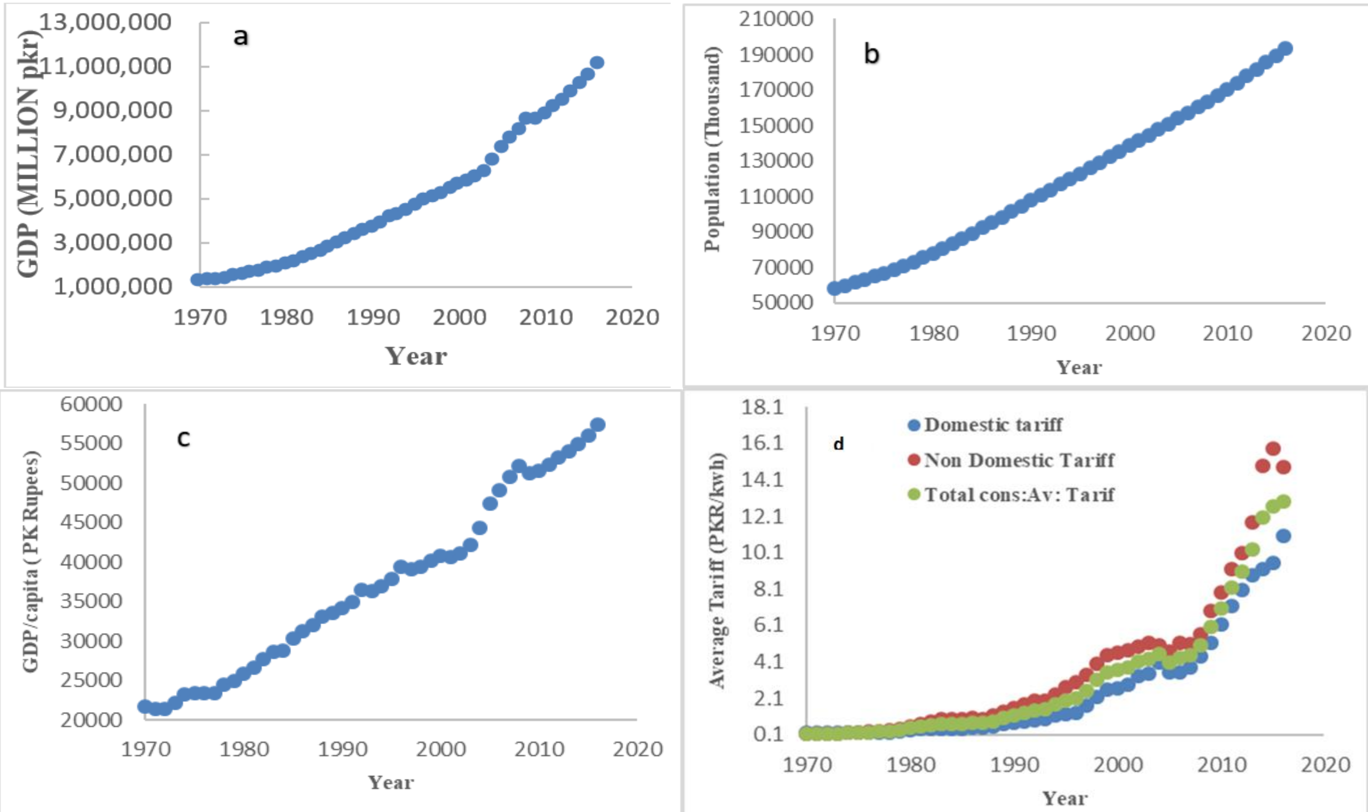

Fig.3.Represents historical data for selected explaining variables (i.e GDP, GDP per capita, population and price)

B. Estimation Of Elasticities

In this part three single equation consumption models are proposed, first one for domestic second for non-domestic and third for total electricity consumption. These models expressed in linear logarithmic form in which annual domestic electricity consumption is linking to annual values of domestic average tariff and GDP per capita for first model, in second model non-domestic consumption links to non-domestic average tariff and GDP, where as in third model total electricity consumption links to total electricity average tariff, GDP and time trend which is proxy for technical progress[12][11][10]

Model 1

$$\text{Log}(E_{dom.t}) = \alpha_o + \alpha_1 \log(X_{1,t}) + \alpha_2 \log(P_t) + \alpha_3 \log(E_{dom,t-1}) \quad (1)$$

Where (Edom, t) is the domestic consumption. X1,t is the per capita GDP. Pt is the price for domestic customers , α0, α1, α2, α3 are coefficients of linear regression and t-i as subscript for the indication of lag term (i.e. t-1 defines lag1.

Model 2

$$\text{Log}\,(\text{E}_{\text{ndom.t}}) = \beta_0 + \beta_1 \log(\text{x}_{2.t}) + \beta_2 \log(\text{PR}_{.t}) + \beta_3 \log(\text{PRN.}_{t-1}) + \beta_4 \log(\text{E}_{\text{ndom},t-1}) \quad (2)$$

Where (Endon, t) is the non-domestic consumption. X2,t is the GDP. PR, t is electricity price for non- domestic customer PRN t is a time trend β0 B1 β2 β3 β4 are regression coefficients.

Model 3

$$\text{Log}\,(E_{total.t}) = \gamma_0 + \gamma_1 \log((X_{3.t}) + \gamma_2 \log(PRT_{.t}) + \gamma_3 \log(PRTN_{,t-1}) + \gamma_4 \log(E_{total.t-1}) \quad (3)$$

Where (E total, t) is the total electricity consumption. X3, t is the GDP. PRT, t is the total electricity average price PRRTNt-3 is a time trend and γ0, γ1, γ2, γ3, γ4 are the regression coefficients.

From equation (1) coefficients α1 α2 represent the price and income elasticities in short-run for domestic electricity usage, and regression coefficients β1 β2 in equation (2) represent the short run price and GDP elasticities for non-domestic electricity usage, whereas from equation (3) regression coefficients γ1 γ2 represent short run GDP and total electricity price for total consumption[12]

From equation (1), (2) and(3) it is expected that short run GDP and GDP per capita (α1, β1, γ1) coefficients should be greater than zero for the reason of economic intensity and to increase the sale of electrical services and goods, where-as short run price elasticities coefficient (α2, β2, γ2)must be less than zero for usual cause economy[2]. Now the elasticities for long-run can be determined by dividing short run elasticities by (1-α3), (1- β4) and (1- γ4) for all above proposed models respectively as mentioned in[12]

$$Lm_1 = \alpha_1/(1-\alpha_3)$$
$$Lm_2 = \alpha_2/(1-\alpha_3)$$

$$Lmn_1 = \beta_1/(1-\beta_4)$$
$$Lmn_2 = \beta_2/(1-\beta_4)$$

$$Lsmc_1 = \gamma_1/(1-\gamma_4)$$
$$Lsmc_2 = \gamma_2/(1-\gamma_4)$$

Lm1 and Lm2 are the long run income and price elasticities for model.1, Lmn1 and Lmn2 are the long run GDP and price elasticities for non-domestic model.2 finally Lsmc1 and Lsmc2 are the long run GDP and total electricity price elasticities for model.3.

The results of estimated co-efficient are presented in Table I, for electricity consumption. Test for serial correlation Breusch-Godfrey serial correlation LM test is applied which notify that residuals are free from serial correlation. Moreover to investigate the presence of unit root and to determine integration order of the selected independent variables Augmented Dickey- Fuller (ADF) test is applied on the natural logarithmic form of variables for three models. (ADF) test statistics presented in Table II, the unit root for null hypothesis could not be rejected at the significance level of 10% only for series PR,t Stationarity is detected at first difference. Again (ADF) at first difference of variable defines that the series PR,t is integrated of order 1, 1(1).Whereas for all other series unit root for null hypothesis cannot be accepted at significance level of 10%, which concludes that they are integrated with order of 0, 1(0) in nature.

TABLE I Statistics summary of elasticities estimations and regression coefficients over the period of 1970-2016 for model 1, model 2 and model 3.

Domestic Model		**Non Domestic Model**		**Total Consumption Model**	
α_0	-20.6844	β_0	-0.27875	γ_0	-1.40868
α_1	5.48389	β_1	0.89937	γ_1	1.18662
α_2	-0.30116	β_2	0.42202	γ_2	0.50639
α_3	0.07019	β_3	-0.48206	γ_3	-0.708
Lm_1	5.89786	β_4	0.34088	γ_4	0.45802
Lm_2	-0.32389	Lmn_1	1.3645	Lmc_1	2.18943
R^2 adjusted	0.968	Lmn_2	0.64028	Lmc_2	0.93435
F	363.689	R^2 adjusted	0.98	R adjusted	0.9929
		F	1120.5	F	1126.24

TABLE II Summary of unit root test "Augmented Dickey-Fuller ADF" on the logarithm of selected variables. The level of confidence at which hypothesis can be rejected that the series contains unit root is illustrated in parenthesis

Variable	ADF test statistics	Test equation
$E_{dom,\,t}$	0.031927 (99.55%)	Constant +Trend
$E_{ndon,\,t}$	-0.490537 (98.06%)	Constant +Trend
$E_{total,\,t}$	-0.084871 (99.37%)	Constant +Trend
$X_{1,t}$	-0.216986 (92.88%)	Constant
$X_{2,t}$	-0.502680 (97.99%)	Constant +Trend

P_t	1.117882 (99.71%)	Constant
$PR,_t$	-1.298541 (87.59%)	Constant +Trend
PRT_t	0.297423 (91.72%)	Constant

C. *Multiple Linear Regression Models*

For the visual analysis of annual electricity consumption up-to 2040, a multiple linear regression model is used, in which we focused annual population and GDP time series. Due to low elasticity impact of price is eliminated from model, as same technique is utilizes in [2]

Constructed models are illustrated as below.

$$E_{dom,t} = \alpha + \beta_1 X_{1,t} + \beta_2 X_{2,t} + \beta_3 E_{dom,t-1} + e \quad (4a)$$

$$E_{ndom,t} = \alpha + \beta_1 X_{1,t} + \beta_2 X_{2,t} + \beta_3 X_{2,t-1} + \beta_4 X_{1,t-1} + \beta_5 E_{ndom,t-1} + e \quad (4b)$$

$$E_{total,t} = \alpha + \beta_1 X_{1,t} + \beta_2 X_{2,t} + \beta_3 X_{1,t-1} + \beta_4 E_{total,t-1} + e \quad (4c)$$

Where Edom, t, Endom, t, Etotal, t are the domestic , non-domestic and total, annual electricity consumption in GWh, X1,t denotes annual GDP in million Pak: rupees, X2,t represents the population in thousands and α, β1, β2, β3, β4, and β5 are the regression coefficients and e is the random error.

The independent variable GDP (X1,t) and Population (X2,t) are forecasted by simple linear regression equations shown below for the time t.

$$X_{1,t} = d_1 + p_1 t \quad (5)$$

$$X_{2,t} = d_2 + p_2 t \quad (6)$$

d_1, d_2, p_1, p_2, are the regression coefficients.

However it is desirable to note that time t is equal to 1970. Equation 5 and 6 are introduced to forecast GDP and Population in order to forecast annual electricity consumption.

For the simplification of first model here another model is proposed on the basis of GDP per capita means ratio between GDP and population as independent variable which is illustrated as a simple linear regression in following equations.

$$E_{dom,t} = \alpha + \beta_1 X_{3,t} + \beta_2 E_{dom,t-1} + e \quad (7a)$$

$$E_{ndom,t} = \alpha + \beta_1 X_{3,t} + \beta_2 E_{ndom,t-1} + e \quad (7b)$$

$$E_{total,t} = \alpha + \beta_1 X_{3,t} + \beta_2 E_{total,t-1} + e \quad (7c)$$

E, t is the electricity consumption in GWh, X3,t is the annual GDP/capita or (income) in Pak: rupees and α, β1 β2 are the regression coefficient and e is the random error. Independent variable GDP per capita X2, t is generated by simple regression as in equation (5), (6)

$$X_{3,t} = d_1 + p_3 t \quad (8)$$

d_3 and p_3 are the coefficients.

Table III describes the matrix of correlation for all selected variables in Eqs (4a)-(4c) data for the period of 1970-2016, there is high degree of correlation between all the dependent and independent variables.

The Breusch-Godfrey Serial Correlation LM test is used to investigate the correlation between dependent and explanatory variables for Eqs (4a)-(4c) and Eqs (7a)-(7c) which shows that there is no serial correlation. For the unit root problem (ADF) test is applied to determine either the data is stationary or not on time series data for the period of 1970-2016 of considered variables (Edom,t, Endom,t, Etotal,t,, X1,t, X2,t, X3,t) and a unit root problem was detected means that the data is non-stationary. For making data stationary differentiated once and ADF test was applied again on the first difference so the variables becomes stationary meaning that there is no any unit root problem detected after first difference as explained in Table IV. Therefore it is decided that they are integrated with order of 1, 1(1)

TABLE III: Matrix of correlation for considered variables of Eq, (4a-4c)

	$E_{dom,t}$	**$E_{ndom\ t}$**	**$E_{total,t}$**	**$X_{1,t}$**	**$X_{2,t}$**
$E_{dom,t}$	1			0.9956	0.988
$E_{ndom\ t}$		1		0.9928	0.99
$E_{total,t}$			1	0.9962	0.9912
$X_{1,t}$				1	0.98786
$X_{2,t}$					1

TABLE IV: Augmented Dickey- Fuller (ADF) test results of considered variables on the first difference. The series contain a unit root can be rejected at high confidence level reported in parenthesis.

Variables	**ADF test statistic**		**Test equations**
$E_{dom,t}$	-4.1062	(>95%)	Constant
$E_{ndon,t}$	-4.6889	(>95%)	Constant
$E_{total,t}$	-4.5649	(>95%)	Constant
$X_{1,t}$	-2.5112	(>95%)	Constant
$X_{2,t}$	-3.1743	(>95%)	Constant
$X_{3,t}$	-4.6344	(>95%)	Constant

According to Engle and Granger if the selected variables are 1,1(1) as given in [12] then Eq.(4)-(7) can be calculated by ordinary least square (OLS) and resulting residuals are stationary, 1(0) then the (Edom,t, Endom,t, Etotal,t, X1,t, X2,t, X3,t) selected variables are said to be co-integrate. Hence the estimation equations are fine in equilibrium relation for long-term estimations.

Augmented Engle and Granger co-integration test is applied on residuals [29][30] of Eq. (4)-(7) conformed that the residuals of all equations are 1(0) and the hypothetical guess of unit root is unacceptable at high confidence level greater than (95%) as defined in Table V. Hence the equations which are estimated to forecast electricity consumption are valid and selected variables are co-integrated. All the regression coefficients are represented in Table VI. Fig:4 shows the residuals of Eq.(4a-4c)-(7a-7c) are all fit inside the average range of (±3%) and there is no revealing a particular behavior, conformed that there is no serial correlation.

TABLE V: Summary of Augmented Engle Granger test

Residuals	ADF test statistic	ADF 95% critical value
Eq. (4a)	-5.1156	-2.9282
Eq.(4b)	-4.5262	-2.9435
Eq.(4c)	-4.7457	-2.9282
Eq.(7a)	-5.2249	-2.9282
Eq.(7b)	-5.1555	-2.9381
Eq.(7c)	-5.4729	-2.9281

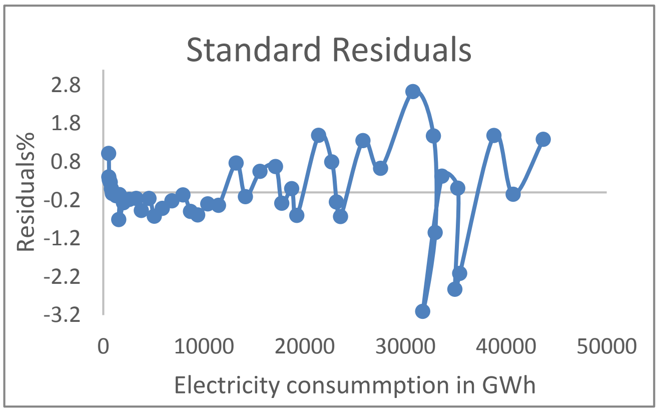

fig: 4 Residual plots of Eq, (4a)

D. Models On Percentage Difference Data Of Year To Year

In this sub section an alternative method is developed in which it is necessary to make the data stationary thereby obtaining the desired forecasting equations. There is a technique to differentiate the data for stationary data series as seen in pervious sub section. First derivative of variables will be involved in the resulting regression equations. On the other hand in a simple way equations will be difficult to handle with derivatives. The technique developed here is to create percentage difference on year to year of variables, as resulting equations can be utilized to measure the percentile deviations with respect to previous year which may be used to perform forecasting. The proposed technique explained in following form.

$$\Delta E_{dom,t} = \alpha + \beta_1 \Delta X_{1,t} + \beta_2 \Delta X_{2,t} + \beta_3 \Delta E_{dom,t-1} e \quad (9a)$$

$$\Delta E_{ndom,t} = \alpha + \beta_1 \Delta X_{1,t} + \beta_2 \Delta X_{2,t} + \beta_3 \Delta X_{2,t-1} + \beta_4 \Delta X_{1,t-1} + \beta_5 \Delta E_{ndom,t-1} + e \quad (9b)$$

$$\Delta E_{total,t} = \alpha + \beta_1 \Delta X_{1,t} + \beta_2 \Delta X_{2,t} + \beta_3 \Delta X_{1,t-1} + \beta_4 \Delta E_{total,t-1} + e \quad (9c)$$

Where Δ is the operator of percentage change (i.e. $\Delta E_{dom,t} = (\Delta E_{dom,t} - \Delta E_{dom,t-1}) / \Delta E_{dom,t}$) while the variables are same as defined in Eq. (4) and (7). The same variables also considered when the GDP per capita plays role in in electricity consumption leading to:

$$\Delta E_{dom,t} = \alpha + \beta_1 \Delta X_{3,t} + \beta_2 \Delta E_{dom,t-1} + e \quad (10a)$$

$$\Delta E_{ndom,t} = \alpha + \beta_1 \Delta X_{3,t} + \beta_2 \Delta E_{ndom,t-1} + e \quad (10b)$$

$$\Delta E_{total,t} = \alpha + \beta_1 \Delta X_{3,t} + \beta_2 \Delta E_{total,t-1} + e \quad (10c)$$

Whereas $\Delta X_{1,t}$, ΔX_{1t-1} and $\Delta X_{3,t}$ can be calculated be same equations (5), (6) and (8).

To determine the correlation in data series the Breusch-Godfey Serial Correlation LM test is applied on Eq. (9a)-(9c) and (10a)-(10c) which shows the residuals are free from serial correlation. The coefficients of Eq. (9) and (10) are presented in Table VIII. The hypothesis of all first differenced variables cannot be accepted at high confidence level more than (99%) except $\Delta X_{2,t}$ whose null hypothesis may be rejected at (95%) as shown in Table VII.

TABLE VI: Regression Coefficients summary of Eq. (4a)-(4c)-(7a)-(7c)

Model	α	β_1	β_2	β_3	β_4	β_5	D	p
Eq.(4a)	-2936.5396	0.866893	0.0284628	0.7496265				
Eq.(4b)	-2043.7572	5.3187987	-0.0087681	0.0520273	-5.3134733	0.8631775		
Eq.(4c)	-7824.1547	2.0572025	0.1701899	-0.064503	0.6531664			
Eq.(5)							-212.71698	211.07351
Eq.(6)							47067.588	2989.187
Eq.(7a)	-3781.9854	0.171353	0.8894045					
Eq.(7b)	-12057.1	0.6328497	0.541173					
Eq.(7c)	-14609.013	0.7101226	0.7400402					
Eq.(8)							17749.171	806.05336

TABLE VII: Unit root test Augmented Dickey-Fuller (ADF) on the considered variables in Eq. (9a-9c) and Eq. (10a-10c) the hypothesis that the series contain unit root can be rejected at confidence level reported in parenthesis.

Variables	ADF test statistic		Test equations
$\Delta E_{dom,t}$	-4.3518	(>99%)	Constant
$\Delta E_{ndon,t}$	-5.9565	(>99%)	Constant
$\Delta E_{total,t}$	-5.7819	(>99%)	Constant
$\Delta X_{1,t}$	-5.8652	(>99%)	Constant
$\Delta X_{2,t}$	-3.6144	(>95%)	Constant + Trend
$\Delta X_{3,t}$	-5.4092	(>99%)	Constant

E. *Root Mean Square Error (RMSE) Analysis*

For visual error analysis, based on root mean square error (RMSE), is a mathematical expression which is provided in order check the model reliability and performance. The standard error can be calculated by following equation.

$$S_{XE}=\sqrt{\frac{\sum_{i=1}^{N}(E-Eest)^2}{N}} \quad (11)$$

Where E is the actual value and E, $_{est.}$ is the is the value of estimation from proposed models.

The model reported by Eq.(4a)-(4c) is within average standard residual of ±3.0% regarding actual time series data, where as the standard residuals for the Eq. (7a)-(7c) ranges between the ±2.0% to ±2.5%. also the similar behavior is detected for Eq. (10a)-(10c).Now for checking the validity of all proposed models in this paper electricity consumption is estimated for 5 years assuming the base year is 2011 so that as in[2]. Moreover for justification the modeling efforts, results are compared with Naïve forecast (i.e. simple regression over the time). As the Table IX, shows all the results of estimated equations with deviations which are somehow greater than acceptable range, but if we compare with Naïve forecasting (i.e. fit line) resulted deviations are close to similar or within acceptable ranges, even simple regression has a good accuracy. Also the validation results for non-domestic and total consumption are detected similar, which are omitted for simplification. It can be noted that equations to forecast electricity consumption with acceptable deviation ranges having good accuracy to the actual data, one can notice that the deviations for the year 2014-2016 are greater due to the average increasing rate of consumption is lower than the average increasing rate. Accordingly it is necessary to consider annual the time series data available (i.e. 1970-2016) to develop the equation for future projection, now we can say that proposed equations in this study are valid models for forecasting Pakistan's electricity consumption

III. RESULTS AND DISCUSSION

A. *Price And GDP Elasticities*

Price and income elasticity are estimated from Eqs. (1), (2) and (3) showed that the higher income elasticity and low price elasticity to the consumption. Short run elasticities are lower than the long-run elasticities as expected. For domestic model short-run and long-run price and income elasticity are calculated by using Eq. (1) short-run and long-run price elasticities are -0.30116 and -0.32389 respectively quit low which does not affect electricity consumption or in other word consumption cannot regulated by varying the electricity price in Pakistan, where-as income elasticities for short-run and long-run are 5.48389 and 5.89786 respectively has higher value which describes that electricity consumption is closely attached to income of domestic costumer but price policies cannot regulate domestic consumption. For example if income doubles, the consumption of electricity increase by nearly 589%. Such that between the period 1980-1992 if domestic customer's income increases nearly 102% as the corresponding increase in domestic consumption was 570% which reacts an similar increase to the estimated elasticity. An estimations for two other non-domestic and total consumption models short-run and long-run price GDP elasticity are reported in table I.

Where-as domestic consumption represents higher value of price elasticity which can be explained with their major flexibility in the use of electricity. Users may react to save energy in case of expensive bills. For instance turning off all unnecessary appliances or make their use for less time (i.e. air conditioning in summer season and electric heater in winter season). As in [31] other passible ways seems quite limited for Pakistan's domestic customers, it is considerable fact that most of the cooking and heating system are dependent on natural gas or LPG (Liquid Petroleum Gas). An-other possibility is the installation of photovoltaic plants which is the alternative of electricity generation for domestic customers, but the initial costs with government contribution are still high.

There are many limitations to save electricity for non-domestic customer, but some of the passible options are more difficult. For example the replacement of old electrical machinery and other appliances with new which require extra cost, may result to decrease profit over investment ratio which may difficult for any businessmen.

Increasing price elasticity by lunching new tariffs with different categories between working days and weekend, also between working hours and late night [32] may be fruitful for non-domestic users, if a businessmen could change his working plan for production and store of goods. Eventually extra salary problem must be in consideration, due to weekends and non-working hours in the trade-off between electricity savings. Smart metering information system also helps non-domestic consumers to reduce price of electricity.

TABLE VIII: Regression co-efficient summary of Eq. (9a-9c) and (10a-10c)

Model	α		β1		β2		β3		β4		β5	
Eq.(9a)	-0.0038	-0.59%	-0.088	-1.46%	0.44573	-1.60%	0.97033	-45.06%				
Eq.(9b)	-0.0755	-2.18%	1.452055	3.90%	5.62541	2.94%	-3.13493	-1.93%	0.1043	0.27%	-0.1927	-1.30%
Eq.(9c)	-0.1012	-3.14%	1.329419	3.97%	6.38951	3.65%	-2.13944	-1.45%	0.2181	0.62%	-0.3012	-2.07%
Eq.(10a)	0.04124	2.57%	0.884258	1.78%	0.3341	2.45%						
Eq.(10b)	0.02833	2.54%	1.267065	3.27%	-0.0582	-0.41%						
Eq.(10c)	0.03835	3.28%	1.200596	3.18%	-0.0224	-0.16%						

Table IX: Presented equations are validated only on domestic electricity consumption model,(the percentage changes with respect to historical data are reported in parenthesis) and compared with a naïve forecasting. The values are in GWh.

Year	Eq(4a)		Eq(7a)		Eq(9a)		Eq(10a)		Naïve forecast	
2012	36745.6	(-4.94%)	36672	(-4.75%)	34715.58	(-0.61%)	34716.67	(-(0.61%)	35229	(-0.85%)
2013	36931	(-4.11%)	36542.9	(-3.09%)	35964.59	(-1.53%)	35981.08	(-1.58%)	34928.6	(-1.38%)
2014	37747.5	(-2.82%)	37149.1	(-4.47%)	42188.38	(-8.00%)	42260.56	(-8.15%)	35412	(-9.6%)
2015	40765.5	(-0.092%)	40358.1	(-0.91%)	42657.5	(-4.52%)	42708.93	(-4.63%)	38813	(-4.93%)
2016	42725.1	(-2.32%)	42295.8	(-3.36%)	46672.89	(-6.32%)	46749.86	(-6.48%)	40728	(-7.34%)

B Forecasts Of Electricity Consumption

Electricity consumption forecasts determined on the basis of linear regression equations compared with the NTDC forecasts results [4] and with an-other National forecasts presented by [14]The forecasts presented by National Transmission and Dispatch Company (NTDC) represents, the official statistical report of Pakistan's Ministry of energy. Their resultant co-efferent are on the basis of microeconomic model which considers the historical data of major sectorial GDP, Population, yearly customer class and selling price/kwh as independent variables for long term planning up to 2037

NTDC provides major contribution in power sector of Pakistan also in cross-border interconnections of Pakistan with south Asia regional countries. They provided national forecasts on the basis of multiple linear regressions

Total electricity consumption forecasts presented in fig; 5(a). All proposed equations declares good agreement with the historical data given in [4] where-as smaller deviations accruing between forecasting equations.

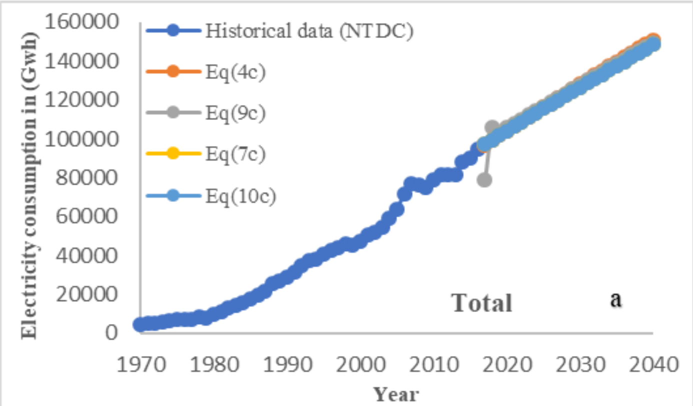

Figure 5 (a)

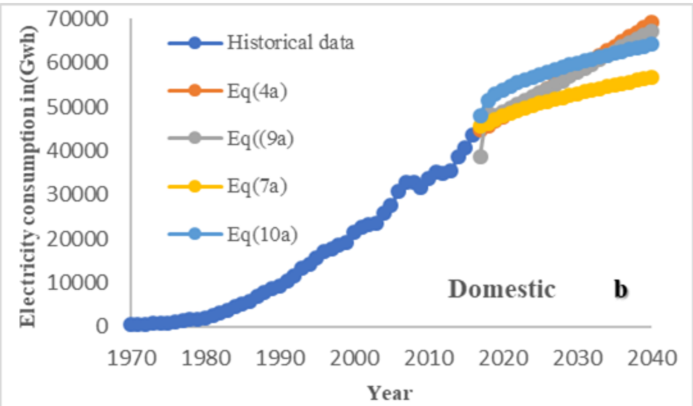

Figure 5 (b)

Fig. 5(b) represents the domestic electricity consumption forecasts, clearly demonstrate that Eq.(4a) and Eq.(9a) seem to in good agreement with data [27][4] where-as Eq.(7a) and Eq.(10a) lead to underestimate nearly 18% and 7% in 2040 respectivelyIt is necessary to mention here in Eq(7a) and Eq.(10a) the explanatory variable is GDP/capita only, where-as in Eq.(4c) and (9c) the considered explaining variables are GDP and population.

Non-domestic electricity consumption forecasts reported in fig; 5(c) nearly same deviations identified where Eq. (7b) and Eq.(10b) results underestimation about 7% and 13% in 2040. While Eq.(4b) and Eq.(9b) leads to perfectly fits the data given in [4].

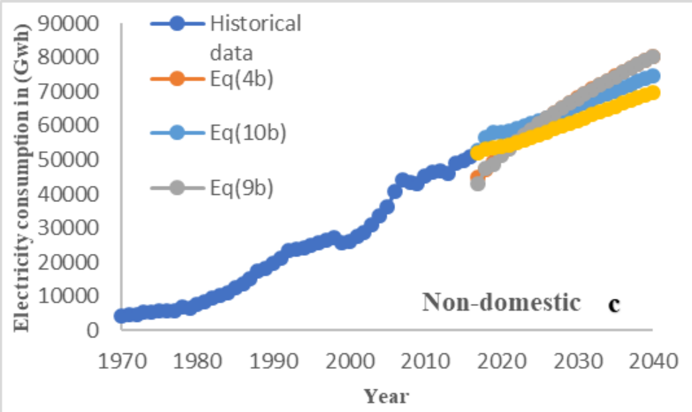

Figure 5 (c)

Fig; 5 (a), (b), (c) forecasts of electricity consumption

The meaning full result achieved from the comparison is that there is good similarities among the available forecasts of Pakistan[14] and proposed equations in this study, also difference between proposed models presents considerable deviations. In this study forecasting time period about 23 years is considered on the behalf of previous 45 years historical data. Prepared equations also has an-other important feature is that they are simple statistical models because there is only fundamental data of explaining variable is needed as input, permits to remove the cast linked to data searching which is basic need for econometric model[2]

Finally it is our personal opinion that the accuracy of proposed models could be increased by better forecasts of explaining variables such as accurate data of population projection available from Pakistan Bureau of Statistics [27] Moreover the accurate future forecasts of GDP could be avail from the State bank of Pakistan, ministry of finance or from merchant banks.

CONCLUSION

In the first part of study, analysis of price elasticities of non-residential and total electricity usage are quit limited, and price elasticities of domestic model results positive value which is greater than other two models. This analysis conclude two main ideas, Electricity price may not be considered as independent variable in forecasting models, for future projection of electricity consumption in Pakistan. And there is no any importance of pricing policies of national electric power regulatory authority (NEPRA) to make optimum usage of electricity in Pakistan. The response of GDP and GDP per capita growth rate leading main role in electricity consumption, changing growth rate of GDP has proportionality to consumption, this relationship was detected from the estimation of elasticities. Therefore there is need to consider these variables as explaining variable for electricity consumption forecasts in Pakistan. In the second part of this paper different regression models were proposed for long-tern future projection of electricity in Pakistan. An increment in total electricity consumption will be result of both domestic and non-domestic consumption in the next years, would be expected with an average growth rate of about 2% per year in Pakistan. According to the data we can easily conclude that the eq.(4c) for total electricity consumption model, and Eq.(9a) and Eq.(4b) for other two models are valid with good accuracy for future projection of domestic and non-domestic consumption respectively, because these equations fit the data accurately. It is noticed that the analysis of elasticities, forecasts and our ideas presented in this study will be fruit full for policy makers and energy planers to build future scenario of electricity consumption in Pakistan.

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