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Segmentation of Images with Inhomogeneous Intensity Multi-Objects

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***Abstract*—**Computer vision is an influential area in which methodologies are generated to analyze and know about the charactristics and construction of a digital image and output is some meaningful information. Image processing comprises five main branches i.e image segmentation, image denoising, image registration, image inpainting and image deblurring. Image segmentation is our focus research work in context of fuzzy sets theory. The pivotal element to fuzzy sets [11] is fuzzy membership V, which acts like region descriptor, must satisfy the restriction $0 \leq u \leq 1$. Level set method (LSM) [9] is used, which is responsible to distribute and allogate the evolution curve C, which is a better way to carry out image segmentation process.

In our research work we developed a model for segmenting images with inhomogeneous intensity multi objects background having maximum, minimum, average intensities. For such achievement we changed Krinidis and Chartiz [13] fitting term by linear term in fuzzy setup. Experimental result of our model justify that our model will show better performance in those images which are suffering from intensity inhomogeneity multi objects.

Keywords: Variational model, Fuzzy sets, Image segmentation, Intensity inhomogeneity.

I. Introduction

Image segmentation is one of the busy research area of image processing. The aim and objective of image segmentation is to separate image domain into different meaningful regions, based on some similar characteristics (color, intensity, texture etc). For this purpose different technique such as thresholding [4], clustering [5], edge detection have been proposed. However, due to presence of noise, intensity inhomogeneity, outlier in images, stable and robust image segmentation is still a challenging task. In variational framework, Active contour models play a vital role for segmentation of images.

Different researchers try to upgrade image segmentation techniques. In particular, Mumfared and Shah [6], presented a well known variational segmentation model. The goal of MS energy functional is to segment a given image Z_0 into different regions in terms of intensity, color or texture etc. The energy function of MS is follow:

$$F(U,K) = \eta.\, length(K) + \int_\Omega |Z_0 - U|^2 dxdy + \int_\Omega |\nabla U|^2 dxdy, \quad (1)$$

where $\Omega \subset R^2$ represent image domain, K is the set of edges, η and ξ are positive parameters. This model solves image segmentation and image denoising model simultaneously. But the drawback of the model is that it is very difficult to implement.

Chan and Vase (CV) [7] proposed a region based model, which is the piece wise constant approximation of the Mumford and Shah model. They have used level set technique [9] for the minimization of the Mumford and Shah model functional [6] The energy functional of the model is follow:

$$F(c_1, c_2, C) = \mu.\, length(\mathrm{C}) + \nu.\,\mathrm{area}(\mathrm{inside(C)}) + \eta_1 \int_\Omega |Z_0 - c_1|^2 dxdy + \eta_2 \int_\Omega |Z_0 - c_2|^2 dxdy, \quad (2)$$

where μ, η_1, η_2, and ν are fixed positive parameters. c_1 *and* c_2 are the average intensity inside and outside of the contour C respectively. The first term in Eq.(2) is the length term make the curve smooth and the last two terms represent the fitting terms or data terms, which guide the contour towards object boundary.

CV model gives bitter result where intensity homogeneity objects in an image but may not segment images with inhomogeneous multi objects.

Wu and He [8] proposed a convex variational level set model based on the coefficient of variation which is proposed by Badshah et al. [10]. Which is strictly convex. For minimization, following is the energy functional of the proposed model:

$$E_{WH}(\phi) = \lambda \int_\Omega \frac{(Z_0(x,y)-a_1)^2(\phi(x,y)+1)^2}{(a_1)^2} dxdy + \int_\Omega \frac{(Z_0(x,y)-a_2)^2(\phi(x,y)-1)^2}{(a_2)^2} dxdy, \quad (3)$$

Where η is a positive tuning parameter, ϕ is a level set function [9], $\bar{a}_1$, $\bar{a}_2$ are two constant prototypes to inner and outside of the contour which is defined in the following way.

If the exterior and interior of the level set function ϕ is non empty, Then

$$a_1 = \frac{\int_\Omega (Z_0)^2(x,y) H(\phi(x,y)) dxdy}{\int_\Omega (Z_0)(x,y) H(\phi(x,y)) dxdy}, \quad (4)$$

$$a_2 = \frac{\int_\Omega (Z_0)^2(x,y)(1-H(\phi(x,y))) dxdy}{\int_\Omega (Z_0)(x,y)(1-H(\phi(x,y))) dxdy}, \quad (5)$$

Otherwise

$$a_1 = a_2 = \frac{\int_\Omega (Z_0)^2(x,y) dxdy}{\int_\Omega (Z_0)(x,y) dxdy}, \quad (6)$$

Here $H(\phi)$ is a Heaviside function is defined in the following way:

The minimization of the energy functional is given by the following Gradient descent flow.

$$\frac{\partial \phi}{\partial t} = -\lambda \frac{(Z_0(x,y)-\bar{a}_1)^2}{(\bar{a}_1)^2}(\phi(x,y)+1) - \frac{(Z_0(x,y)-\bar{a}_2)^2}{(\bar{a}_2)^2}(\phi(x,y)-1) \quad (7)$$

CV model is strictly convex and free of initial contour placement. This model can segment images with homogeneous intensity multi object images successfully and might not gives better result in multi-variate intensity object images.

Ali et al. [12] proposed a novel region based model in which they used Generalized averages combining with non Euclidean measure. The energy function of the proposed model is as under:

$$E(G_{v1}, G_{v2}, u) = \mu \int_\Omega [u(x,y)]^m (1(-\hat{k}(Z_0(x,y), G_{v1})))dxdy + \int_\Omega [1-u(x,y)]^m (1-(\hat{k}(Z_0(x,y), G_{v2})))dxdy \quad (8)$$

Where μ is a +ve parameter, $u(x,y)$ is a fuzzy membership function defined in (4) which is defined on [0,1]. The fuzzy function is used to reject local minima. m is a weighting exponent on each fuzzy membership function u, which is usually taken 2. This model is convex. This model work very well in those images which have noise, pepper and salt, intensity homogeneity. But this model may not work very well in those images which have sever intensity inhomogeneity multi objects. From experimental result we can easy guess that this model does not work the mentioned limitation.

Fuzzy sets [11], which have been widely used in data clustering and image segmentation is firstly introduce by Krinidis and Chatzis [1], unification active contour methodology and fuzzy sets [11]. They proposed a model fuzzy energy based active contour model [1]. For minimization they proposed energy fuctional is follow:

$$F(C, \bar{a}_1, \bar{a}_2) = \eta_1 \int_\Omega [u(x,y)]^m (Z_0 - \bar{a}_1)^2 dxdy + \eta_2 \int_\Omega [1-u(x,y)]^m (Z_0 - \bar{a}_2)^2 dxdy. \quad (9)$$

Where u(x,y) is fuzzy function which is defined on [0,1] and m is a weighting exponent usually taking 2. $\bar{a}_1$, $\bar{a}_2$ are the average prototype inside and outside of Γ. They incorporate fuzzy sets in the active contour symmetry. This model uses pseudo zero level set as the dynamic curve. The fuzziness of the energy function provides strong ability to reject local minima. The relation between pseudo level set and the evolving curve Γ is defined in the following way:

$$\begin{cases} \Gamma = \{(x,y) \in \Omega : u(x,y) = 0.5\} \\ \Gamma_1(inside\Gamma) = \{(x,y) \in \Omega : u(x,y) > 0.5\} \\ \Gamma_2(outside\Gamma) = \{(x,y) \in \Omega : u(x,y) < 0.5\}. \end{cases} \quad (10)$$

The fuzzy membership function u(x,y) is updated by:

$$u(x,y) = \frac{1}{1+\left(\frac{\lambda_1(Z_0-\bar{a}_1)^2}{\lambda_2(Z_0-\bar{a}_2)^2}\right)^{\frac{1}{m-1}}} \quad (11)$$

FEBAC gives successful result in those images having homogeneous intensities with multiple objects and might be unsuccessful in inhomogeneous intensity multiple objects.

II. PROPOSED MODEL

In this section our propose model will be discussed briefly. The main idea behind our proposed model is to segment a digital image having variate intensity multi objects. Let us assume the dynamic curve Γ in the image domain Ω, and the image is denoted by Z_0. This model is based on linear fitting terms and pseudo level set (fuzzy membership function). The energy functional of our proposed model is as under:

$$E(L_1, L_2, u) = \mu \int_\Omega |\nabla u(x,y)| dxdy + \int_\Omega |Z_0 - L_1|^2 [u(x,y)]^m dxdy + \int_\Omega |Z_0 - L_2|^2 [1-u(x,y)]^m dxdy. \quad (12)$$

Where $u(x,y)$ is a fuzzy membership function in (11), which must satisfy the constraint $0 \le u \le 1$, $\mu>0$ is a +ve parameter and $L_1 = a_0 + a_1x + a_2y$, s $L_2 = b_0 + b_1x + b_2y$ are linear terms. The fuzziness of the energy functional provide strong ability to deny the local minima. We will get the global minimum value of the energy functional in (12), because the model is convex. Next we will prove the convexity of the proposed energy functional.

Convexity of the energy functional

For simplicity consider the energy functional in (12) as follow:

$$f(\zeta) = \epsilon f1(\zeta) + f2(\zeta) + f3(\zeta) \quad (13)$$

$$f_1(\zeta) = \int_\Omega |\nabla u(x,y)| d\zeta \quad (14)$$

$$f_2(\zeta) = \int_\Omega |Z_0 - L_1|^2 [u(x,y)]^m d\zeta \quad (15)$$

$$f_3(\zeta) = \int_\Omega |Z_0 - L_2|^2 [u(x,y)]^{m-1} d\zeta \quad (16)$$

To show that the domain Ω is convex, for this we consider

$$f_2(\zeta) = \int_\Omega |Z_0 - L_1|^2 [u(x,y)]^m d\zeta \quad (17)$$

And let

$$F_2(\zeta) = |Z_0 - L_1|^2[u(x,y)]^m \qquad (18)$$

Such that $F_2: \Omega \longrightarrow$ R, and let

$$f_2(\zeta) = \int_\Omega F_2(\zeta)\, d\,\zeta \qquad (19)$$

First of all we will show that Ω is convex, for this let us consider that $z_1 = (x_1,y_1)$, $z_2 = (x_2,y_2) \in \Omega$ and for any $\gamma \in [0,1]$, we have

$$\gamma z_1 + (1-\gamma)z_2 = (\gamma\,(x_1,y_1) + (1-\gamma)(x_2,y_2)) = (\gamma\,(x_1 - x_2) + x_2, \gamma\,(y_1 - y) + y_2) \in \Omega \qquad (20)$$

Ω is convex as $x_1 - x_2 \in$R and so that $\gamma(x_1 - x_2) + x_2 \in \Omega$ also $y_1 - y_2 \in R$ and $\gamma \in [0,1]$, therefore $\gamma(y_1 - y_2) + y_2 \in \Omega$. so we can write that $\gamma z_1 + (1-\gamma)z_2 \in \Omega$. Now differentiating eq. (18), w.r.t the fuzzy membership function u , we have

$$\frac{\partial F_2}{\partial u} = m[u(\zeta)]^{m-1}|Z_0 - L_1|^2\,. \qquad (21)$$

Differentiating again w.r.t to u, we get

$$\frac{\partial^2 F_2}{(\partial u)^2} = m(m-1)[u(\zeta)]^{m-2}|Z_0 - L_1|^2 \qquad (22)$$

$\frac{\partial^2 F_2}{(\partial u)^2} \geq 0$, as m=2 obviously m is +ve , $u(\zeta) \in [0,1]$ and $|Z_0 - L_1|^2 \geq 0$. Also Ω is convex, so F_2 is convex and for all ζ_1, $\zeta_1 \in \Omega$ and $\gamma \in [0,1]$ the inequality

$$F_2((\gamma\zeta_1) + (1-\gamma)\zeta_2) \leq \gamma F_2(\zeta_1) + (1-\gamma)F_2(\zeta_2)$$

So from the above inequality, we have

$$\int_\Omega F_2((\gamma\zeta_1) + (1-\gamma)\zeta_2)d\zeta \leq \int_\Omega \gamma F_2(\zeta_1)d\zeta + \int_\Omega (1-\gamma)F_2(\zeta_2)d\zeta$$

So using back eq. (17), we have

$$f_2((\gamma\zeta_1) + (1-\gamma)\zeta_2) \leq \gamma f_2(\zeta_1) + (1-\gamma)f_2(\zeta_2)$$

From the above inequality which is the condition for convexity of a function, we can say that $f_2(\zeta)$ is convex.

In the similar way we can also prove that $f_3(\zeta)$ is convex. Thus $f(\zeta)$ is convex, being the sum of convex functions. So the Energy functional of our proposed model is convex with respective the fuzzy membership function u.

keeping u fixed, and minimizing the energy function $E(L_1, L_2, u)$ with respective L_1 i.e a_0, a_1, a_2 and L_2 i.e b_0, b_1, b_2.

$$\frac{\partial E}{\partial a_0} = 0\,,$$

$$\Rightarrow \int_\Omega Z_0 u^m dxdy - a_0 \int_\Omega u^m dxdy - a_1 \int_\Omega x u^m dxdy - a_2 \int_\Omega y u^m dxdy = 0 \qquad (23)$$

$$\frac{\partial E}{\partial a_1} = 0$$

$$\Rightarrow \int_\Omega Z_0 x u^m dxdy - a_0 \int_\Omega x u^m dxdy - a_1 \int_\Omega x^2 u^m dxdy - a_2 \int_\Omega xy u^m dxdy = 0, \qquad (24)$$

$$\frac{\partial E}{\partial a_2} = 0$$

$$\Rightarrow \int_\Omega Z_0 y u^m dxdy - a_0 \int_\Omega y u^m dxdy - a_1 \int_\Omega xy u^m dxdy - a_2 \int_\Omega y^2 u^m dxdy = 0 \qquad (25)$$

We can solve the above equation for a_0, a_1, a_2 using matrix inversion method or Cramer rule.

Now minimizing the energy equation with respective L_2 i.e b_0, b_1, b_2

$$\frac{\partial E}{\partial b_0} = 0$$

$$\Rightarrow \int_\Omega Z_0 u^m dxdy - b_0 \int_\Omega u^m dxdy - b_1 \int_\Omega x u^m dxdy - b_2 \int_\Omega y u^m dxdy = 0 \qquad (26)$$

$$\frac{\partial E}{\partial b_1} = 0 \Rightarrow \int_\Omega Z_0 x u^m dxdy - b_0 \int_\Omega x u^m dxdy - b_1 \int_\Omega x^2 u^m dxdy - b_2 \int_\Omega xy u^m dxdy = 0, \qquad (27)$$

$$\frac{\partial E}{\partial b_2} = 0$$

$$\Rightarrow \int_\Omega Z_0 y u^m dxdy - b_0 \int_\Omega y u^m dxdy - b_1 \int_\Omega xy u^m dxdy - b_2 \int_\Omega y^2 u^m dxdy = 0. \qquad (28)$$

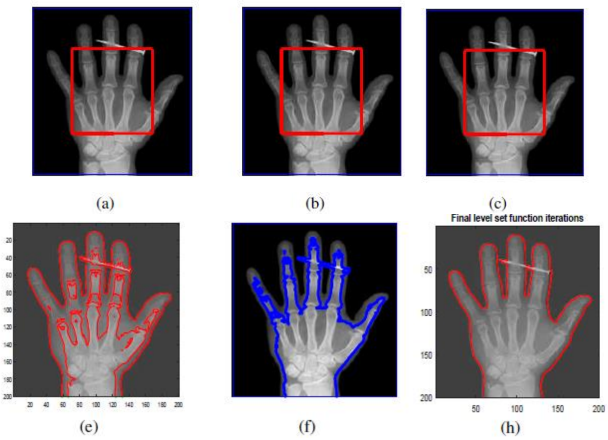
We can also solve above equation for the unknown b_0, b_1, b_2 by using matrix inversion method or Cramer rule.

Keeping L_1 and L_2 fixed and minimizing the energy functional E with respective fuzzy membership function $u(x,y)$, we will get the associated Euler-lagrange equation (evolving equation).

$$\frac{\partial u}{\partial t} = \mu \frac{\nabla u}{|\nabla u|} - m\lambda_1 u^{m-1}|Z_0 - L_1|^2 - m\lambda_2(1-u)^{m-1}|Z_0 - L_2|^2\,. \qquad (29)$$

III. Experimental Results

In this section we exhibits some experimental results of our model comparing with other existing models like CVL [8], FEBAC [1], Ali et al. [12]. For all experiments, image of a window of size 5 × 5. In figure (1) our model in relation to three other existing models has been tested on a real multi variate intensity image.



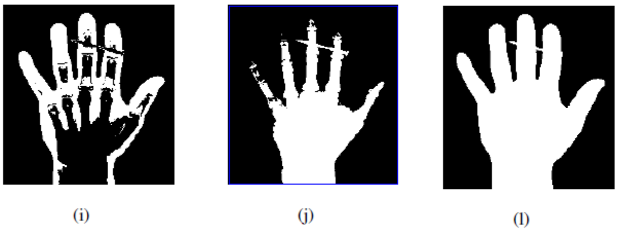


Figure 1: Performance of existing models and our proposed model on variate intensity image. First column shows the performance of Ali et al model [12], second column Krinidis Chartiz [1], third column Wu-He [8] and the fourth column shows the performance of our proposed model respectively (with JS= 0.89 with other parameters $\alpha = 0.18, \sigma = 0.5, \lambda_1 = 0.17, \lambda_2 = 1$)
. Excellent performance of our model in respect to other existing models on inhomogeneous intensity image can be viewed. Our model extract whole object in an image, while other model cannot.

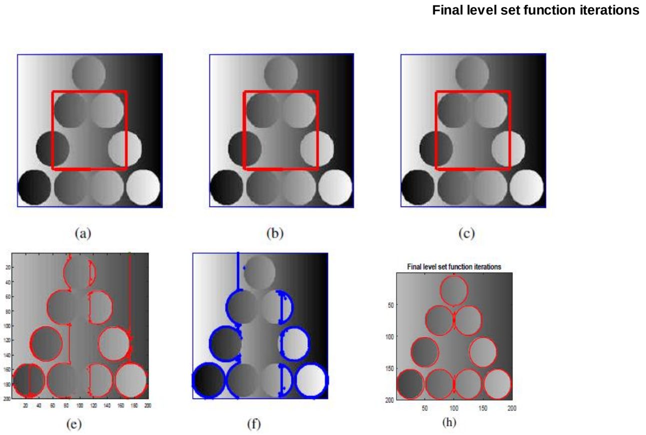


Figure 2: Result of existing models and our proposed model on variate intensity multi object image. First column shows the performance of Ali et al model [12], second column Krinidis Chartiz [1], third column Wu-He [8] and the fourth column shows the performance of our proposed model respectively (with JS= 0.887) with other parameters $\mu = 0.838, \sigma = 0.5, \lambda1 = 0.7, \lambda2 = 1.3$). Excellent performance in respect to other existing models of our model on inhomogeneous intensity multi object image can be observed. Our model segment very well where severe intensity inhomogeneity in the foreground.

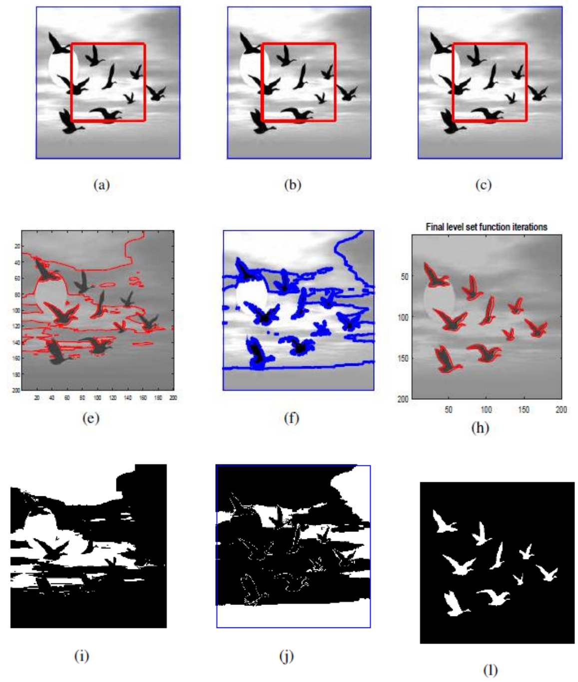
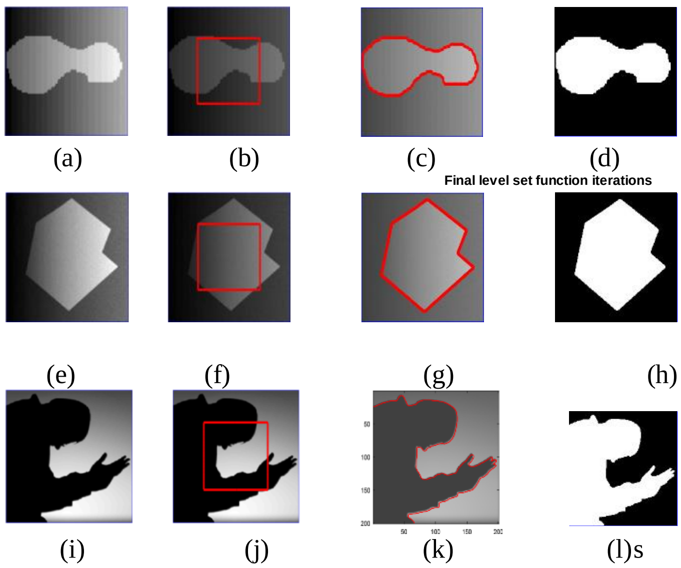
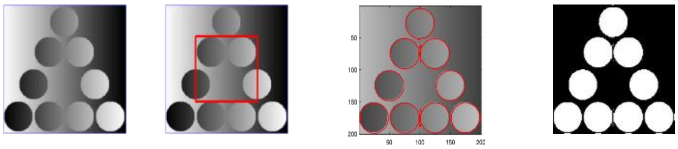
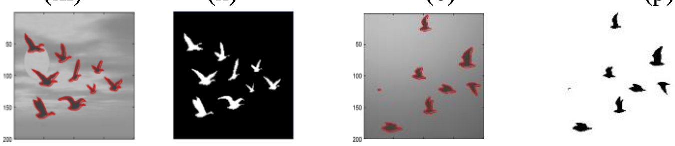


Figure 3: In figure (3), a real image with same intensity objects in the foreground but the background is cluttered. FEBAC model [1] (first column, JS= 0.32), Wu-He model [8] (second column, JS= 0.1), Ali et al. model [12], (third column, JS= 0.23), proposed model (fourth column, JS= 0.97) with other parameters $\mu = 0.01, \lambda1 = 8 \lambda2 = 4$ image having multi objects holding cluttered background. Our proposed model qualitatively behave very good and captured all desired objects and also quantitatively proposed model do better.





(m) (n) (o) (p)



(q) (r) (s) (t)

Figure 4: All these are the experimental results of our proposed model. From the first row (image (a)) it can be easily guess that our model works well in images which have low and average intensity background and foreground. From 3rd row (image (i)) result show that our model also work well in images which have high and low intensity background and foreground. From image (m) which have multiple objects with low, average, and high intensity, our model shows better performance.

CONCUSLION

In this research work, we suggested a new variational model based on fuzzy membership function for segmenting images with intensity inhomogeneity multi objects. Experimental results on some real and synthetic images have showed that our proposed model is more efficient and robust for segmenting images with intensity inhomogeneity, multi-intensity regions and objects. Comparisons with Ali et al. [12] model, FEBAC [1] model and also show the superiority of our proposed model in intensity inhomogeneity. The computational cost of our proposed model is far better than the said models.

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